ՀՀ ԿՐԹՈՒԹՅԱՆ, ԳԻՏՈՒԹՅԱՆ, ՄՇԱԿՈՒՅԹԻ ԵՎ ՍՊՈՐՏԻ ՆԱԽԱՐԱՐՈՒԹՅՈՒՆ
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ՊԵՏՐՈՍՅԱՆ ՏԻԳՐԱՆ ԱՐԱՅԻ

ՈՐՈՇ ՔՎԱՆՏԱՅԻՆ ԵՐԵՎՈՒՅԹՆԵՐ ԿՈՍՄՈԼՈԳԻԱԿԱՆ ՄՈԴԵԼՆԵՐՈՒՄ

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SOME QUANTUM EFFECTS IN COSMOLOGICAL MODELS

Thesis for the degree of Candidate of physical and mathematical sciences
Speciality 01.04.02 - ”Theoretical Physics”

ABSTRACT

YEREVAN - 2022

Ատենախոսության թեման հաստատվել է Երևանի պետական համալսարանում

Գիտական ղեկավար՝	ֆիզ.-մաթ. գիտ. դոկտոր, պրոֆեսոր Ա.Ա. Սահարյան
Պաշտոնական ընդդիմախոսներ՝	ֆիզ.-մաթ. գիտ. դոկտոր, պրոֆեսոր Ս.Դ. Օդինցով ֆիզ.-մաթ. գիտ. դոկտոր, պրոֆեսոր Գ.Ս. Հաջյան
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The thesis theme is approved at the Yerevan State University

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The defense of the thesis will take place at 12:00 on 29 April 2022 on the session of the Specialized Council 049 Physics of the Yerevan State University.

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The thesis is available in the library of the Yerevan State University.

The abstract is distributed on 19 March 2022.

Scientific secretary of the Specialized Council	Candidate of Phys. Math. Sciences, Associate Prof. V.P. Kalantaryan

GENERAL DESCRIPTION OF THE WORK

Relevance of topic. Currently the theory for quantum gravity is not available and the interaction of the gravitational field with quantum matter is considered within the framework of semiclassical theory where the gravitational field is considered as a classical background. It is expected that the effects of quantum gravity will be essential on Planck scales and this approach has wide range of applications. In particular, the investigations of quantum field theoretical effects in gravitational fields have resulted in the formation of the research field known as quantum field theory in curved spacetimes. Among the interesting effects in that research area are the vacuum polarization and the creation of particles by strong gravitational fields. These effects have important implications in the physics of the early universe and in the physics of black holes. Interesting applications of quantum field theory on curved backgrounds have recently appeared in condensed matter physics. In constructing quantum field theories in general background geometries, among the most important points is the selection of a physically meaningful vacuum state. The vacuum state depends on the choice of complete set of mode functions used in the second quantization procedure. Those functions, and consequently the properties of the vacuum state, are sensitive to both the local and global characteristics of background spacetime. Already for the Minkowski bulk, depending on the spacetime coordinates corresponding to different observers, different vacuum states are realized. An example is the Fulling-Rindler vacuum that presents the vacuum state for a uniformly accelerated observer different from the standard Minkowskian vacuum of inertial observers. The corresponding coordinates cover only a part of Minkowski spacetime (right and left Rindler wedges). Other patches of Minkowski spacetime inside the future and past light cones correspond to the so-called Milne universe. The Milne universe is described by Friedmann-Robertson-Walker (FRW) type line element with negative curvature spatial sections and with the scale factor being a linear function of the time coordinate. It is well adapted for considerations of different types of the vacuum states in backgrounds with time-dependent metric tensors (see, for example, [1] and references therein).

Another important topic of quantum field theory in curved spacetime is the investigation of quantum effects in homogeneous and isotropic cosmological backgrounds. Due to the high symmetry of the corresponding geometry, a relatively large number of problems are exactly solvable, and the corresponding results may shed light on the influence of the gravitational field on quantum fields for more complicated geometries. The expectation value of the energy-momentum tensor for quantum fields may break the energy conditions appearing in the formulations of the Hawking-Penrose singularity theorems and the quantum effects of nongravitational fields may provide a way to solve the cosmological singularity problem. During the inflationary expansion in the Early Universe, the quantum fluctuations of fields are responsible for the generation of density perturbations serving as seeds for the large scale structure formation in the Universe. In inflationary models (for a review see [2]) the cosmological expansion is described by an approximately de Sitter (dS) spacetime. The short phase with this type of accelerated expansion provides a natural solution for a number of problems in Standard Cosmology. On the other hand, the Universe at recent epoch is accelerating. The corresponding expansion is well approximated by a model where the mean energy density of the Universe is dominated by a positive cosmological constant. With this source as a dark energy, the dS spacetime appears as a future attractor for the large scale geometry of the Universe. Consequently, the investigation of physical effects in dS background geometry is of interest for understanding both the early Universe and its future. Another motivation is related to the holographic duality between quantum gravity on dS spacetime and a quantum field theory living on its timelike infinity. Among the simplest cosmological backgrounds allowed by string theories is the linearly expanding spatially flat cosmological model. Recently various aspects of quantum field theory in a linearly expanding universe have been an object of intense investigations. Among the most interesting effects allowing a comprehensive study are the vacuum polarization and particle production by the time-dependent gravitational field.

The aim of the thesis is to investigate the local characteristics of the vacuum state for different bulk and boundary geometries and the dependence of the corresponding properties on the choice of the vacuum state. We have considered:

- The vacuum expectation values (VEVs) of the field squared and of the energy-momentum tensor for a scalar field in the Milne patch of the Minkowski spacetime in the presence of a spherical boundary.
- The two-point functions and local characteristics for the scalar vacuum in a linearly expanding cosmological background in the geometry of two planar boundaries. The physical nature of the vacuum forces acting on the plates and their dependence on the boundary conditions imposed on the field operator.
- The combined effects of background gravitational field and of a spherical boundary on the local properties of the scalar vacuum in de Sitter spacetime.
- The dependence of the properties for scalar and fermionic fields in de Sitter spacetime on the choice of the vacuum state and the conformal relation with different vacuum states in Minkowski spacetime.

Scientific novelty. The scalar field modes are specified for different vacuum states in the Milne universe. Closed analytic expressions are provided for the Hadamard function and for the VEVs of the field squared and energy-momentum tensor inside and outside a spherical boundary for the general case of Robin boundary condition. The vacuum energy-momentum tensor has an off-diagonal component that describes an energy flux along the radial direction. The Casimir effect for a scalar field is considered in the geometry of two planar boundaries on background of FRW cosmological model with a scale factor being a linear function of the time coordinate. An integral representation of the Hadamard function is provided with an explicit separation of the boundary-induced contributions. That is important from the point of view of the renormalization of the local VEVs in the coincidence limit. For points near the plates, the dominant contribution to the VEVs comes from the vacuum fluctuations with short wavelengths and the effects of gravity on the boundary-induced mean field squared and on the diagonal components of the vacuum energy-momentum tensor are weak. The influence of the gravitational field on those VEVs is essential at distances from the plates larger than the curvature radius of the spacetime. Qualitatively new feature of the cosmological expansion is the appearance of nonzero off-diagonal component of the vacuum energy-momentum tensor that corresponds to the energy flux along the direction perpendicular to the plates. The direction of that flux depends on the values of the coefficients in the boundary conditions. The Casimir forces acting on the plates are investigated. The boundary induced effects on the local VEVs of a scalar field with general curvature coupling are studied for the hyperbolic (H-) vacuum in dS spacetime. As a boundary, a spherical shell is considered with a constant comoving radius in hyperbolic coordinates. The corresponding complete set of mode functions are constructed without specifying the vacuum state. It is shown that the conformal and adiabatic vacua coincide. As local characteristics of the H-vacuum, the VEVs of the field squared and of the energy-momentum tensor are studied. An interesting feature is the presence of the vacuum energy flux along the radial direction. The energy flux can be directed either from the sphere or towards the sphere. In early stages of the expansion the effects of the spacetime curvature on the sphere-induced VEVs are weak and, to the leading order, they coincide with the corresponding VEVs for a sphere in the Milne universe. The effects of gravity are essential at late stages. The fermionic condensate (FC) is investigated in dS spacetime for Bunch-Davies (BD) vacuum state.

Practical importance. The mode functions for a scalar field in different bulk and boundary geometries can be used for the investigation of expectation values of physical observables in more general setups, such as the finite temperature effects and the effects in interacting field theories. The expressions for the two-point functions can be employed for the study of the response of Unruh-de Witt type of particle detectors in a given state of motion. The generalization of the summation formula for series over the zeros of the combination of the associated Legendre function of the first kind and its derivative with respect to the degree can be used in other boundary-value problems of the mathematical physics in background of the spaces with a constant negative curvature. We expect that similar series will appear also in the investigations of the VEVs for fermionic fields. The two-point function for the hyperbolic vacuum in dS spacetime can be used in the investigations of entanglement between two regions of dS spacetime foliated by negative curvature spaces.

Basic results to be defended:

1. Complete set of the modes and the Hadamard function are obtained for a scalar field in background of the Milne universe in the presence of a spherical boundary with Robin boundary condition on it. The sphere-induced contribution in the Hadamard function is explicitly separated. Representations are provided for both the interior and exterior regions which are well-adapted for the investigation of local VEVs. The VEVs of the field squared and of the energy-momentum tensor are decomposed into the boundary-free and sphere-induced contributions. For non-conformally coupled or for massive fields, a new qualitative feature, induced by the sphere, is the appearance of the nonzero off-diagonal component of the vacuum energy-momentum tensor that describes energy flux along the radial direction. Depending on the values of the parameters, the flux can be directed either from the sphere or towards the sphere.

2. For a conformally coupled massive scalar field in background of linearly expanding FRW spacetime and in the presence of two parallel planar boundaries, the Hadamard function is decomposed into the boundary-free and boundary-induced contributions for general case of Robin boundary consitions. In this way the renormalization of the VEVs of physical observables is reduced to the one in the boundary-free geometry. For a massive field the vacuum energy-momentum tensor has an off-diagonal component that corresponds to the energy flux along the direction normal to the boundaries. The energy flux can be either positive or negative, depending on the Robin coefficients.

3. Near the plates, the influence of the gravitational field on the VEVs of the field squared and of the diagonal components of the energy-momentum tensor is weak and the leading terms in the corresponding asymptotic expansions coincide with those for Minkowski bulk. The influence of the gravitational field is essential at proper distances from the plates larger than the curvature radius of the spacetime. At large distances the decay

of the VEVs, as functions of the proper distance, is power law for both massless and massive fields. For massive fields that behavior is in clear contrast with the case of the Minkwoski bulk where the decay is exponential. Depending on the boundary conditions and on the separation between the plates, the Casimir forces acting on the plates can be either attractive or repulsive. In the special case of Dirichlet boundary condition on both the plates the forces are attractive. For Neaumann boundary condition the forces can be either attractive or repulsive, depending on time and on the separation.

4. For a scalar field with general curvature coupling parameter, the complete set of mode functions and the Hadamard function are constructed for the H-vacuum in de Sitter spacetime in the presence of a spherical boundary. For the Robin boundary condition on the sphere the eigenvalues for the radial quantum number are specified in the exterior and interior regions. The sphere-induced contributions in the VEVs of the field squared and of the energy-momentum tensor are investigated. In addition to the diagonal components, corresponding to the vacuum energy density and the stresses, the VEV of the energy-momentum tensor has a nonzero off-diagonal component that describes energy flux along the radial direction. Depending on the value of the Robin coefficient and also on the radial coordinate, the flux can be directed either from the sphere or towards the sphere.

5. In early stages of the cosmological expansion the effects of the spacetime curvature on the sphere-induced VEVs are weak and, to the leading order, they coincide with the corresponding VEVs for a sphere in the Milne universe. The effects of gravity are essential at late stages. Depending on the mass of the field quanta, two qualitatively different regimes of the decay for the VEVs are realized. For the values of the mass smaller than the critical value, determined by the curvature radius and the curvature coupling parameter, the decay of the sphere-induced VEVs, as functions of the time coordinate, is monotonic. For masses larger than the critical value, the decay is oscillatory damping with exponentially decreasing amplitude.

6. For points near the sphere the dominant contribution to the VEVs comes from the modes with large values of the angular momentum. The influence of the gravitational field on those modes is weak and the leading terms in the expansions of the VEVs for the field squared and for the energy density and azimuthal stresses coincide with those for a spherical boundary in flat spacetime with the distance from the sphere replaced by the proper distance in dS bulk. Near the sphere the VEVs of the field squared, the energy density and azimuthal stresses have the same sign in the exterior and interior regions, whereas the energy flux and radial stress have opposite signs. In the exterior region, at large distance from the sphere the VEVs decay exponentially for both massive and massless fields.

Approbation of the work. The results of the thesis were reported at the conferences "Modern Physics of Compact Stars and Relativistic Gravity" (Yerevan, Armenia, 2019, 2021), Yerevan State University Student Scientific Society 5th International Conference (Yerevan, Armenia, 2018) and have been discussed at the seminars of the Chair of Theoretical Physics of Yerevan State University.

Publications. Seven papers are published on the topic of the thesis.

Structure of the thesis. The thesis consists of Introduction, three Chapters, Conclusion and the list of references. It contains 196 pages, including 26 figures.

CONTENT OF THE THESIS

In **Introduction** the scientific literature related to the topic of the thesis is reviewed, the relevance of the topic is argued, the aim of the work, the scientific novelty and the practical value are presented, the basic results to be defended are described.

In **Chapter 1** the local properties of the conformal (C-) vacuum for a massless scalar field in the Milne universe are investigated. The line element for the $(D+1)$-dimensional Milne universe is expressed as (system of units with $c=1$, $\hbar=1$ is used)

$$ds^2 = dt^2 - t^2(dr^2 + \sinh^2 r d\Omega_{D-1}^2), \tag{1}$$

where for the time and dimensionless radial coordinates one has $0 \le t < \infty$, $0 \le r < \infty$, and $d\Omega_{D-1}^2$ is the line element on a $(D-1)$-dimensional sphere with unit radius. The spatial part corresponds to a constant negative curvature space covered by the hyperspherical coordinates (r, ϑ, ϕ). For the set of angular coordinates we have $\vartheta = (\theta_1, \theta_2, \ldots \theta_n)$, $0 \le \theta_k \le \pi$, $k = 1, 2, \ldots, n$ and $0 \le \phi \le 2\pi$, where $n = D-2$.

The spacetime described by the line element (1) is flat, however the spatial geometry corresponds to a negatively curved space. We consider a quantum scalar field $\varphi(x)$ with $x = (t, r, \vartheta, \phi)$ and with curvature coupling parameter ξ. The field equation reads

$$\left(\Box + m^2 + \xi\mathcal{R}\right)\varphi = 0, \tag{2}$$

where $\Box$ is the d'Alembert operator and for the background under consideration the Ricci scalar is zero, $\mathcal{R} = 0$. The normalizable modes are given by the expression

$$\varphi_\sigma(x) = b_1 \frac{X_{iz}(mt)}{t^{(D-1)/2}} \frac{P_{iz-1/2}^{-\mu}(u)}{\sinh^{D/2-1} r} Y(m_p; \vartheta, \phi), \ \mu = l + D/2 - 1, \tag{3}$$

where $u = \cosh r$, σ stands for the set of quantum numbers specifying the solutions, $P_\nu^\rho(y)$ is the associated Legendre function and $Y(m_p; \vartheta, \phi)$ are the hyperspherical harmonics. The function $X_{iz}(mt)$ can be expressed in terms of Hankel or Bessel functions as

$$X_{iz}(mt) = c_1 H_{iz}^{(1)}(mt) + c_2 H_{iz}^{(2)}(mt) = d_1 J_{-iz}(mt) + d_2 J_{iz}(mt). \tag{4}$$

The coefficient b_1 is obtained from the normalization condition. For the adiabatic and conformal vacuum states one has $c_1 = 0$ and $d_2 = 0$, respectively. Substituting the mode functions into the mode-sum formula the corresponding Wightman function for the Milne universe is obtained. The result is specified for a massless scalar field prepared in the C-vacuum. We are interested in the difference of the local characteristics of the C- and Minkowski vacua. The differences in the VEVs of the field squared, $\Delta\langle\varphi^2\rangle = \langle\varphi^2\rangle_{\mathrm{C}} - \langle\varphi^2\rangle_{\mathrm{M}}$, and energy-momentum tensor are obtained from the corresponding difference of the Wightman functions in the coincidence limit. The difference in the VEVs of the energy-momentum tensor is given by

$$\Delta\langle T_i^k\rangle = \mathrm{diag}\left(-1, \frac{1}{D}, \ldots, \frac{1}{D}\right) \frac{2^{1-D}\pi^{-D/2}}{\Gamma(D/2)t^{D+1}} \int_0^\infty dy \, \frac{y^{D-2}A_D(y)}{e^{2\pi y} + (-1)^D} \left[y^2 + D(D-1)(\xi_D - \xi)\right], \tag{5}$$

where $A_D(y) = \prod_{l=0}^{D/2-2} \left[(l+1/2)^2/y^2 + 1\right]$ for even $D \ge 4$ and $A_D(y) = \prod_{l=0}^{(D-3)/2} (l^2/y^2 + 1)$ for odd $D \ge 3$, and $\xi_D = (D-1)/(4D)$ is the curvature coupling parameter for a conformally coupled field. Note that the VEV of the energy-momentum tensor is traceless for non-conformal coupling as well.

In Section 1.3 the influence of a spherical boundary on the local properties of the vacuum is investigated for a massive scalar field in the bulk described by (1). We assume the presence of a spherical boundary with radius r_0 on which the scalar field obeys Robin boundary condition

$$\left(A - \delta_{(\mathrm{j})} B \partial_r\right) \varphi(x) = 0,\ r = r_0, \tag{6}$$

where A and B are dimensionless constants, $\mathrm{j} = \mathrm{i}, \mathrm{e}$, with $\delta_{(\mathrm{i})} = 1$ for the interior region and $\delta_{(\mathrm{e})} = -1$ for the exterior region. Special cases $B = 0$ and $A = 0$ correspond to Dirichlet and Neumann boundary conditions. Inside the sphere, $r < r_0$, the regular mode functions realizing the C-vacuum have the form (3) with a different normalization coefficient. The corresponding eigenvalues of radial quantum number z are determined by the boundary condition (6). They are the roots for the equation

$$\bar{P}^{-\mu}_{iz-1/2}(u_0) = 0,\ u_0 = \cosh r_0. \tag{7}$$

For a given function $f(x)$, the barred notation in (7) is defined in accordance with

$$\bar{f}(x) = [A\sqrt{x^2-1} + (D/2-1)\delta_{(\mathrm{j})} B x] f(x) - \delta_{(\mathrm{j})} B (x^2-1) f'(x). \tag{8}$$

The corresponding positive solutions are denoted by $z = z_k$, $k = 1, 2, \ldots$. The set of quantum numbers is specified by $\sigma = (k, m_p)$. The radial part of the corresponding mode functions in the exterior region, $r > r_0$, is written as a linear combination of the associated Legendre functions $P^{-\mu}_{iz-1/2}(u)$ and $Q^{-\mu}_{iz-1/2}(u)$. One of the coefficients in that combination is determined by the boundary condition (6) and the second one is found from the normalization condition. The mode-sums are evaluated for the Hadamard two-point functions $G(x, x') = \langle 0|\varphi(x)\varphi(x') + \varphi(x')\varphi(x)|0\rangle$ inside and outside the sphere. They are decomposed as $G(x, x') = G_0(x, x') + G_{\mathrm{b}}(x, x')$, where $G_0(x, x')$ is the Hadamard function in the boundary-free geometry and $G_{\mathrm{b}}(x, x')$ is induced by the sphere. For the interior region, the summation of the series over the roots z of the equation (7) is done by using the generalized Abel-Plana summation formula [3,4]. The VEVs of the field squared and energy-momentum tensor are obtained from the Hadamard function and its derivatives in the coincidence limit of the arguments. That limit is divergent and a renormalization is required. For points away from the sphere the divergences are contained in the boundary-free part only. The VEVs are presented as $\langle \varphi^2 \rangle = \langle \varphi^2 \rangle_0 + \langle \varphi^2 \rangle_{\mathrm{b}}$ and $\langle T_i^k \rangle = \langle T_i^k \rangle_0 + \langle T_i^k \rangle_{\mathrm{b}}$, where the parts with the index 0 are the renormalized VEVs in the boundary-free geometry for the C-vacuum. For the region $r < r_0$ the sphere-induced contribution in the mean field squared is given by

$$\left\langle \varphi^2 \right\rangle_{\mathrm{b}} = -\sum_{l=0}^{\infty} \frac{e^{-i\mu\pi} D_l}{S_D t^{D-1}} \int_0^{\infty} dx\, x \frac{\bar{Q}^{\mu}_{x-1/2}(u_0)}{\bar{P}^{-\mu}_{x-1/2}(u_0)} \frac{F^{(\mathrm{i})}_{\mu}(t, r, x)}{\sin(\pi x)}, \tag{9}$$

where $S_D = 2\pi^{D/2}/\Gamma(D/2)$ is the surface area of the sphere with unit radius in D-dimensional space and $D_l = (2l+n)\,\Gamma(l+n)/[(D-1)!l!]$ is the degeneracy of the angular mode with given l. In (9), the notation

$$F^{(\mathrm{i})}_{\mu}(t, r, x) = J_x(mt) J_{-x}(mt) \frac{[P^{-\mu}_{x-1/2}(u)]^2}{\sinh^{D-2} r}, \tag{10}$$

is introduced. The expressions for the diagonal components $\langle T_k^k\rangle_{\mathsf{b}}$ (no summation over $k = 0, 1, \ldots D$) are obtained from the right-hand side of (9) by the replacement $F_\mu^{(\mathrm{i})}(t, r, x) \to t^{-2}\hat{F}_{(k)}F_\mu^{(\mathrm{i})}(t, r, x)$, where $\hat{F}_{(k)}$ are second order differential operators with respect to t and r. The vacuum energy-momentum tensor has a nonzero off-diagonal component $\langle T_0^1\rangle = \langle T_0^1\rangle_{\mathsf{b}}$ that describes energy flux along the radial direction. The appearance of the flux is purely sphere-induced effect and the expression for $\langle T_0^1\rangle_{\mathsf{b}}$ is obtained from (9) replacing $F_\mu^{(\mathrm{i})}(t, r, x)$ by $t^{-3}\left[D\left(\xi_D - \xi\right) + \left(\xi - 1/4\right)t\partial_t\right]\partial_r F_\mu^{(\mathrm{i})}(t, r, x)$. The energy flux vanishes for a conformally coupled massless field. The expression for the sphere-induced VEVs $\langle\varphi^2\rangle_{\mathsf{b}}$ and $\langle T_i^k\rangle_{\mathsf{b}}$ in the region $r > r_0$ are obtained from those in the interior region by the replacements $\bar{Q}_{x-1/2}^{\mu}(u_0) \rightleftarrows \bar{P}_{x-1/2}^{-\mu}(u_0)$ and by the replacement $P_{x-1/2}^{-\mu}(u) \to Q_{x-1/2}^{\mu}(u)$ in the expression (10) for $F_\mu^{(\mathrm{i})}(t, r, x)$. In the absence of the spherical boundary the geometry is homogeneous and the renormalized VEVs $\langle\varphi^2\rangle_0$ and $\langle T_i^k\rangle_0$ depend on the time coordinate only.

In the early stages of the expansion, $t \to 0$, to the leading order one has $\langle\varphi^2\rangle_{\mathsf{b}} \approx (a/t)^{D-1}\langle\varphi^2\rangle_{\mathsf{b}}^{(\mathrm{st})}$ and $\langle T_k^k\rangle_{\mathsf{b}} \approx (a/t)^{D+1}\langle T_k^k\rangle_{\mathsf{b}}^{(\mathrm{st})}$, where $\langle\varphi^2\rangle_{\mathsf{b}}^{(\mathrm{st})}$ and $\langle T_k^k\rangle_{\mathsf{b}}^{(\mathrm{st})}$ are the corresponding VEVs for a massless field in static spacetime with a negative constant curvature space [5]. The latter depend on the radial coordinate alone. The energy flux density behaves as $\langle T_0^1\rangle \propto 1/t^{D+2}$ for a non-conformally coupled field ($\xi \neq \xi_D$) and as $\langle T_0^1\rangle \propto 1/t^D$ in the case $\xi = \xi_D$. At late stages of the expansion and for massive fields, assuming that $mt \gg 1$, the VEVs exhibit damping oscillatory behavior. The leading terms in the expansions of the vacuum stresses are isotropic and decay like $\langle T_k^k\rangle_{\mathsf{b}} \propto \sin(2mt)/t^D$ for $k = 1, \ldots, D$. The decay of the energy density is stronger, like $\langle T_0^0\rangle_{\mathsf{b}} \propto \cos(2mt)/t^{D+1}$, and the energy flux behaves as $\langle T_0^1\rangle_{\mathsf{b}} \propto \cos(2mt)/t^{D+2}$. The late-time asymptotic for $\langle\varphi^2\rangle_{\mathsf{b}}$ contains two parts. The first one is monotonically decreasing as $1/t^D$ and the behavior of the second one is damping oscillatory, as $\sin(2mt)/t^D$.

In Figure 1, for the $D = 3$ Milne universe and for a conformally coupled field, the boundary-induced VEVs in the energy (left panel) and energy flux (right panel) densities are displayed as functions of the time coordinate. The full and dashed curves correspond to Dirichlet and Robin boundary conditions. For the latter we have taken $\beta = -0.6$. The graphs are plotted for $r_0 = 2$ and the numbers near the curves are the values of the radial coordinate r.

Near the sphere the total VEVs $\langle\varphi^2\rangle$ and $\langle T_i^k\rangle$ are dominated by the sphere-induced contributions $\langle\varphi^2\rangle_{\mathsf{b}}$ and $\langle T_i^k\rangle_{\mathsf{b}}$. The leading terms in the expansions of $\langle\varphi^2\rangle$, of the energy density and the tangential stresses coincide with those for a sphere in the Minkowski bulk where the distance from the sphere is replaced by the proper distance $t|r - r_0|$ for the geometry (1). Those terms behave as $\langle\varphi^2\rangle_{\mathsf{b}} \propto 1/(t|r - r_0|)^{D-1}$ and $\langle T_k^k\rangle_{\mathsf{b}} \propto 1/(t|r - r_0|)^{D-1}$, $k = 0, 2, \ldots, D$. For the normal stress and the energy flux one gets $\langle T_1^1\rangle_{\mathsf{b}} \propto (r_0 - r)\langle T_0^0\rangle_{\mathsf{b}}$ and $\langle T_0^1\rangle \propto (r_0 - r)\langle T_0^0\rangle_{\mathsf{b}}/t$. For a minimally coupled field and near the sphere the energy flux in the interior region is directed from the sphere for Dirichlet boundary condition and towards the sphere for non-Dirichlet boundary conditions. For a conformally coupled field the leading terms vanish and the divergences on the sphere are weaker. At large distances from the sphere, the radial dependence of the leading terms in the sphere-induced VEVs is given by $e^{(1-D)r}/r$ and they are exponentially suppressed for both massive and massless fields.

In **Chapter 2** the scalar Casimir densities and forces are considered for the geometry of two parallel plates in background of a linearly expanding spatially flat $(D + 1)$-dimensional cosmological model. The latter is described by the line element

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2,\ a(t) = bt, \tag{11}$$

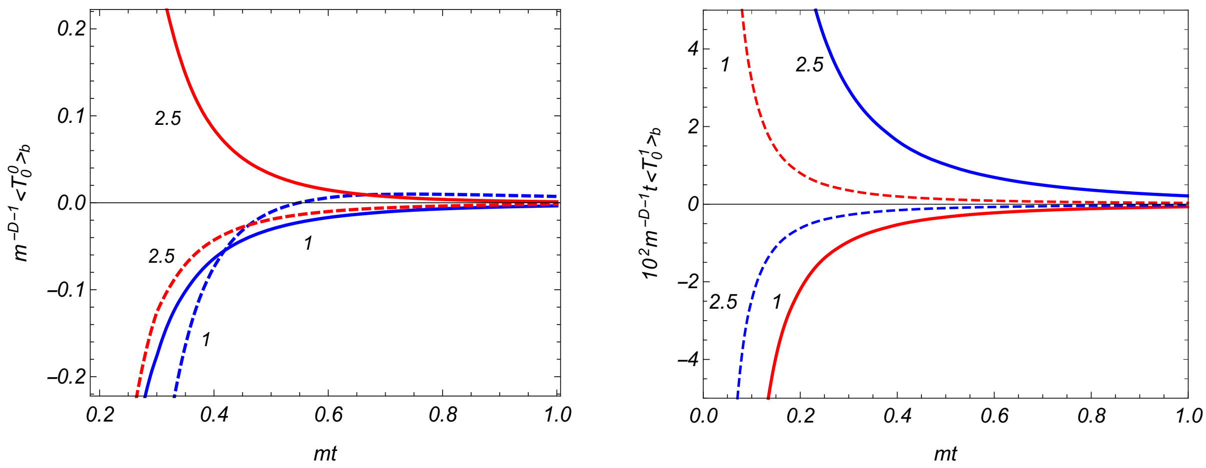

Figure 1: The sphere-induced energy density (left panel) and energy flux density (right panel) for $D = 3$ conformally coupled scalar field as functions of the time coordinates. The graphs are plotted for $r_0 = 2$ and the numbers near the curves present the values of the radial coordinate. The full and dashed curves correspond to Dirichlet and Robin (with $\beta = -0.6$) boundary conditions, respectively.

with spatial coordinates $\mathbf{x} = (x^1, x^2, \ldots, x^D)$. In (11), $0 \leqslant t < \infty$ and $b > 0$ is a constant having dimension of inverse length. The setup is described in Section 2.1. A massive scalar field $\varphi(x)$ with the curvature coupling parameter ξ is considered. The corresponding field equation is given by (2) with $\mathcal{R} = D(D-1)b^2 e^{-2b\eta}$. Two flat boundaries (plates) are located at $x^D \equiv z = z_1$ and $z = z_2$, $z_2 > z_1$. On the plate $z = z_j$, $j = 1, 2$, the field operator is constrained by the boundary condition $[1 + (-1)^{j-1}\beta_j \partial_z]\varphi = 0$, where β_j, $j = 1, 2$, are constants. The complete set of mode functions and the adiabatic and conformal vacua are specified in Sections 2.2-2.3. By evaluating the mode-sum, in Section 2.4 the Hadamard function for the region between the plates is presented in the form of the sum of boundary-free and plate-induced contributions. An integral representation for the latter is provided well adapted for evaluation of the local VEVs.

As important local characteristics of the vacuum state, the VEVs of the field squared and energy-momentum tensor have been considered in Sections 2.5-2.6. They are decomposed into the boundary-free and boundary-induced parts. The corresponding analytical expressions are provided for the region between the plates, $z_1 < z < z_2$, and for the regions $z < z_1$ and $z > z_2$. As a consequence of the time dependence of the background geometry, the boundary-induced VEV of the energy-momentum tensor has a nonzero off-diagonal component corresponding to the energy flux along the direction normal to the plates. The effects of the gravity are crucial at distances from the plates larger than the curvature radius. The decay of the boundary-induced VEVs, as functions of the distance from the plate, is power law for both massless and massive fields. For a massless field the problem is conformally related to the corresponding problem with two parallel plates in the Minkowski bulk. In this special case the off-diagonal component vanishes and the boundary-induced contribution in the VEV of the energy-momentum tensor is traceless. The trace anomaly is present in the boundary-free part only.

The vacuum pressure on the plates is decomposed into the self action and interaction contributions. The latter is induced by the presence of the second plate. Because of the homogeneity of the background spacetime, the self action parts are the same on the left- and right-hand sides of the plates. As a consequence, the corresponding net force becomes zero and the Casimir forces are conditioned by the presence of the second plate. The force per unit surface acting on the plate at $z = z_j$ is given by

$$P_j = -\frac{2^{1-D}\pi^{(1-D)/2}}{\Gamma(\frac{D-1}{2})\,(z_0 b)^D\, t^{D+1}} \int_0^\infty du\, u^{D-1} \frac{2\,(u/bz_0)^2 + [2 + c_j(u) + 1/c_j(u)]\,\hat{f}_D}{c_1(u)c_2(u)e^{2u} - 1} S_D\left(mt, \frac{u}{bz_0}\right), \tag{12}$$

where $z_0 = z_2 - z_1$, $c_j(u) = (\beta_j u/z_0 - 1)\,/\,(\beta_j u/z_0 + 1)$ and $\hat{f}_D = -\left(t^2\partial_t^2 + t\partial_t\right)/\,(4D)$. The function in the integrand of (12) is defined as

$$S_D(mt, x) = \int_0^1 ds\, s(1 - s^2)^{(D-3)/2} \frac{J_{xs}(mt) J_{-xs}(mt)}{\sin\left(\pi x s\right)}. \tag{13}$$

For a massless field $S_D(mt, x) = \Gamma((D-1)/2)/[2\sqrt{\pi}\Gamma(D/2)x]$ and one gets $P_j = P_j^{(\mathrm{M})}/\,(bt)^{D+1}$, where $P_j^{(\mathrm{M})}$ is the corresponding pressure for plates in the Minkowski bulk with the separation z_0. The latter is the same for both plates regardless the values of the coefficients β_j. At small separations between the plates, compared with the curvature radius of the background spacetime, one has $bz_0 \ll 1$ and to the leading order $P_j \approx P_j^{(\mathrm{M})}/\,(bt)^{D+1}$. In that limit the effects of gravity on the Casimir forces are small and the leading term coincides with that in the Minkowski bulk multiplied by the conformal factor. For Dirichlet boundary condition on one plate and for non-Dirichlet boundary condition on the other the forces are repulsive at small separations. In the remaining cases, at small separations the forces are attractive. For separations larger than the curvature radius, $bz_0 \gg 1$, for non-Dirichlet boundary conditions ($\beta_j \neq 0$) one gets $P_j \propto [J_1^2(mt) - J_0^2(mt)]\,/\,(btz_0)^{D-1}$. The corresponding Casimir forces can be either attractive or repulsive. For Dirichlet boundary condition on both plates the leading term behaves as $P_j \propto J_0^2(mt)/\,(btz_0)^{D+1}$ and the forces are attractive.

Figure 2 displays the dependence of the Casimir pressure P_j on mt for a $D = 3$ scalar field with the Neumann boundary condition. The graphs are plotted for fixed values of bz_0 (the numbers near the curves). The latter corresponds to the coordinate distance between the boundaries measured in units of $1/b$. As seen, for a given coordinate separation z_0, at early stages of the expansion one has $P_j < 0$ and the Casimir forces are attractive. At late stages we have an oscillatory damping behavior. The first zero of the force with respect to mt decreases with increasing bz_0.

In **Chapter 3** quantum field-theoretical effects are studied for scalar and fermionic fields on the background of dS spacetime. In Section 3.1, we consider the foliation of $(D + 1)$-dimensional dS spacetime by negative curvature spatial sections. The corresponding coordinates are described and their relations to global and inflationary coordinates are discussed. In Section 3.2 the general solutions of the field equation are found for a massive scalar field with general curvature coupling parameter and various vacuum states are discussed for dS spacetime described by hyperbolic coordinates. The corresponding line element reads

$$ds^2 = dt^2 - \alpha^2 \sinh^2(t/\alpha)\,(dr^2 + \sinh^2 r d\Omega_{D-1}^2), \tag{14}$$

with α being the curvature radius of dS spacetime. The field equation is given by (2) with the Ricci scalar $\mathcal{R} = D(D + 1)/\alpha^2$. The normal modes and Hadamard functions are presented for the hyperbolic (H-) and Bunch-Davies (BD) vacua for a massive scalar field with general

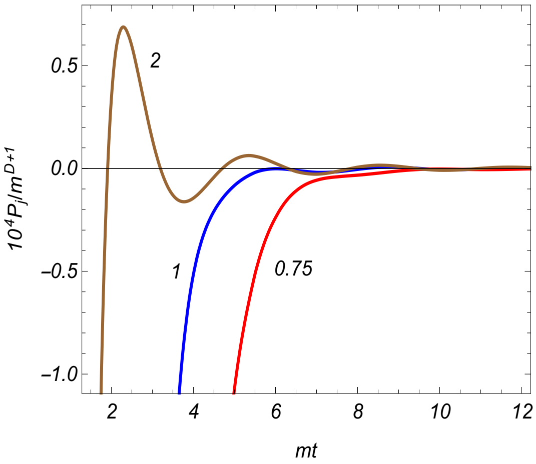

Figure 2: The Casimir pressure as a function of mt for a $D = 3$ scalar field with the Neumann boundary conditions on both plates. The graphs are plotted for fixed values of bz_0 (the numbers near the curves).

curvature coupling parameter. The corresponding expressions are essentially simplified for a conformally coupled massless field. Closed analytic expressions are derived for the VEVs of the field squared and energy-momentum tensor. It is shown that the relations between the VEVs in the H- and BD vacua are similar to those between the C- and Minkowski vacua in flat spacetime. In particular, the difference in the VEVs of the energy-momentum tensor, $\langle T_i^k \rangle - \langle T_i^k \rangle_{\text{BD}}$, is given by formula which is obtained from the righ-hand side of (5) making the replacement $t \to \alpha \sinh(t/\alpha)$.

In Section 3.3 we discuss the effects of a spherical boundary with radius $r = r_0$ on local characteristics of the H-vacuum for a scalar field. On the sphere the field obeys the Robin boundary condition (6). In inflationary coordinates, the boundary under consideration corresponds to an expanding sphere. The mode functions, realizing the H-vacuum inside and outside the sphere, are specified. By using the mode-sum formula, closed analytic expressions are provided for the corresponding Hadamard functions. The sphere-induced contributions are separated explicitly. Based on the results for the Hadamard functions, the VEVs of the field squared and energy-momentum tensor inside and outside the spherical shell are investigated and the corresponding numerical analysis is provided. The sphere-induced contributions in the VEVs of the field squared and energy-momentum tensor, denoted as $\langle \varphi^2 \rangle_{\text{s}}$ and $\langle T_i^k \rangle_{\text{s}}$, are expressed in the form of integrals involving the associated Legendre functions. The problem is inhomogeneous with respect to the time and radial coordinates and the off-diagonal component $\langle T_0^1 \rangle_{\text{s}}$ is different from zero. Similar to the case of a sphere in the Milne universe, it describes energy flux in radial direction. Inside the sphere the corresponding expression reads

$$\langle T_0^1 \rangle_{\text{s}} = \frac{\sinh^{-3}(t/\alpha)}{\alpha^{D+2} S_D} \sum_{l=0}^{\infty} D_l \int_0^{\infty} dx \frac{x e^{-i\mu\pi}}{\sin(\pi x)} \frac{\bar{Q}_{x-1/2}^{\mu}(u_0)}{\bar{P}_{x-1/2}^{-\mu}(u_0)} \times \left[(1/4 - \xi)\left(y^2 - 1\right)\partial_y + \xi y \right] \partial_r F^{(\text{i})}(x, y, u), \tag{15}$$

where the notations (8) and

$$F^{(\mathrm{i})}(x,y,u)=\frac{P_{\nu-1/2}^{x}(y)\,P_{\nu-1/2}^{-x}(y)}{(y^2-1)^{(D-1)/2}}\frac{[P_{x-1/2}^{-\mu}(u)]^2}{(u^2-1)^{D/2-1}} \tag{16}$$

are used. Other notations are defined as $\nu=\sqrt{D^2/4-m^2\alpha^2-\xi D(D+1)}$ and $y=\cosh(t/\alpha)$. The energy flux per unit proper surface area is given by $\langle\tilde{T}_0^1\rangle_{\mathrm{s}}=\alpha\sinh(t/\alpha)\,\langle T_0^1\rangle_{\mathrm{s}}$. The expressions for the diagonal components $\langle T_k^k\rangle_{\mathrm{s}}$ (no summation over k) have a structure similar to (15) with more complicated opertors (containing ∂_y and ∂_r) acting on the function $F^{(\mathrm{i})}(x,y,u)$.

The general formulas for the VEVs are complicated and in order to clarify the qualitative features we have considered limiting cases and various asymptotic regions of the parameters. In the flat spacetime limit, corresponding to $\alpha\to\infty$, the line element (14) is reduced to the line element (1) for the Milne universe. It is checked that, in this limit, from the results for the H-vacuum the corresponding VEVs are obtained for a sphere in the Milne universe, assuming that the scalar field is prepared in the conformal vacuum. For a conformally coupled massless scalar field the problem is conformally related to the problem with a spherical boundary in static spacetime with constant negative curvature space. As another check, it is shown that the VEVs in those problems are connected by the standard conformal relation. In this special case the energy flux vanishes. In early stages of the expansion, corresponding to $t/\alpha\ll 1$, the effects of the spacetime curvature on the sphere-induced VEVs are weak and, to the leading order, they coincide with the corresponding VEVs for a sphere in the Milne universe. The effects of gravity are essential for $t/\alpha\gtrsim 1$. In particular, at late stages, $t/\alpha\gg 1$, the behavior of the VEVs is qualitatively different for positive and purely imaginary values of the parameter ν. For $\nu>0$ the decay of the sphere-induced VEVs, as functions of the time coordinate, is monotonic, as $e^{-(D-2\nu)t/\alpha}$ for $\langle\varphi^2\rangle_{\mathrm{s}}$, $\langle T_k^k\rangle_{\mathrm{s}}$, and like $e^{-(D+1-2\nu)t/\alpha}$ for the energy flux density $\langle\tilde{T}_0^1\rangle_{\mathrm{s}}$. For imaginary ν the decay is oscillatory with the amplitude decreasing as $e^{-Dt/\alpha}$ for diagonal components $\langle T_k^k\rangle_{\mathrm{s}}$ and like $e^{(D+1)t/\alpha}$ for the flux density $\langle\tilde{T}_0^1\rangle_{\mathrm{s}}$.

For points near the sphere the dominant contribution to the VEVs comes from the modes with large values of the angular momentum. The influence of the gravitational field on those modes is weak and the leading terms in the expansions of the VEVs for the field squared and for the energy density and azimuthal stresses coincide with those for a spherical boundary in flat spacetime with the distance from the sphere replaced by the proper distance $\alpha\sinh(t/\alpha)\,|r-r_0|$ in dS bulk. They behave as $|r-r_0|^{1-D}$ for the field squared and as $|r-r_0|^{-D-1}$ for the energy density and azimuthal stresses. Near the sphere these VEVs have the same sign in the exterior and interior regions. The leading terms for the energy flux and radial stress behave like $|r-r_0|^{-D}$ and have opposite signs inside and outside the sphere. The leading terms do not depend on the mass. In the case of the energy-momentum tensor they vanish for a conformally coupled field. In the exterior region, at large distances from the sphere, the sphere-induced VEVs are suppressed by the factor $e^{-(D-1)r}/r$. For a conformally coupled massless field the leading terms vanish and the suppression at large distances is stronger, like $e^{-(D-1)r}/r^2$.

In Figure 3, the boundary-induced energy (left panel) and energy flux (right panel) densities are displayed as functions of the radial coordinate for a minimally coupled scalar field. The graphs are plotted for $m\alpha=t/\alpha=1$, $r_0=1.5$ in the cases of Dirichlet boundary condition and for Robin conditions with $\beta=-3,-0.5$ (the numbers near the curves). Contrary to the energy density, the boundary-induced energy flux density has opposite signs inside and outside the sphere. As seen, the energy flux in the interior and exterior regions is directed from the boundary for Dirichlet boundary condition and towards the boundary for Robin conditions.

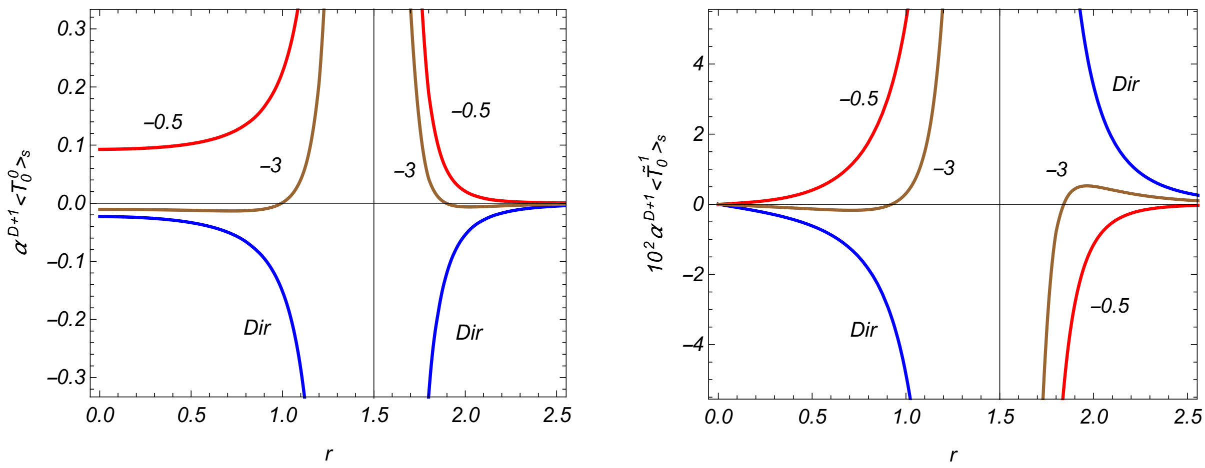

Figure 3: The sphere-induced energy density and the energy flux density as functions of the radial coordinate for $D = 3$ minimally coupled scalar field with Dirichlet and Robin boundary conditions ($\beta = -3, -0.5$). The graphs are plotted for $m\alpha = t/\alpha = 1$, $r_0 = 1.5$.

Section 3.4 is devoted to the investigation of the FC $\langle\bar{\psi}\psi\rangle$ for Dirac fermion field $\psi(x)$ with mass m in background of dS spacetime described by planar coordinates $(t, z^1, \ldots, z^D)$. The corresponding line element is expressed as $ds^2 = dt^2 - e^{2t/\alpha}\sum_{i=1}^{D}(dz^i)^2$. It is assumed that the field is prepared in the maximally symmetric BD vacuum state. As an additional renormalization condition for the renormalized fermionic condensate we require that $\langle\bar{\psi}\psi\rangle_{\text{ren}} \to 0$ in the limit $m \to \infty$. For even values of the spatial dimension D the renormalized FC has the form

$$\langle\bar{\psi}\psi\rangle_{\text{ren}} = \frac{(-1)^{D/2}(4\pi)^{(1-D)/2}N\alpha^{-D}}{2\Gamma((D+1)/2)\,(e^{2\pi m\alpha}+1)}\prod_{l=1}^{D/2}\left[m^2\alpha^2 + (l-1/2)^2\right], \tag{17}$$

with $N = 2^{[(D+1)/2]}$ being the number of spinor components. For odd values of D one has

$$\langle\bar{\psi}\psi\rangle_{\text{ren}} = -\frac{\alpha^{-D}N}{(4\pi)^{\mu}}\frac{(m\alpha)^D}{\Gamma(\mu)}\left\{\sum_{l=1}^{\mu-1}\frac{f_l}{(m\alpha)^{2l}} + 2\left\{\text{Re}[\Psi(im\alpha)] - \ln(m\alpha)\right\}\prod_{l=1}^{\mu-1}\left(1+\frac{l^2}{m^2\alpha^2}\right)\right\}, \tag{18}$$

where $\mu = (D+1)/2$, $\Psi(x)$ is the digamma function and the numerical coefficients f_l are expressed in terms of the Bernoulli coefficients B_{2n}. The expression (17) coincides with the result obtained in [5] by using the point-splitting procedure and the adiabatic subtraction. For a massless field the FC vanishes for odd D and is nonzero for even D. Depending on the number of spatial dimensions the FC can be either positive or negative. The change in the sign may lead to instabilities in interacting field theories. As an example, a system of interacting scalar and fermionic fields with the interaction Lagrangian density proportional to $\varphi^2\bar{\psi}\psi$ is considered. Depending on the value and sign of the FC, the effective mass squared for the scalar field $\varphi(x)$ may become negative.

CONCLUSIONS

1. The VEVs of the field squared and energy-momentum tensor for a massless scalar field are investigated in the Milne universe with general number of spatial dimensions, assuming that the field is prepared in the C-vacuum. An integral representation for the

difference of the Wightman functions corresponding to the C- and Minkowski vacua is derived. The Minkowski vacuum state can be interpreted as a thermal one with respect to the conformal vacuum. The thermal factor is of the Bose-Einstein type in odd dimensional space and of the Fermi-Dirac type in even number of spatial dimensions.

2. The influence of a spherical boundary on the vacuum fluctuations of a massive scalar field is investigated in background of the Milne universe with general number of spatial dimensions, assuming that the field obeys Robin boundary condition on the sphere. For the C-vacuum, the Hadamard function is decomposed into boundary-free and sphere-induced contributions and an integral representation is obtained for the latter in both the interior and exterior regions. As important local characteristics of the vacuum state the VEVs of the field squared and of the energy-momentum tensor are investigated. The vacuum energy-momentum tensor has an off-diagonal component that corresponds to the energy flux along the radial direction.

3. The quantum vacuum effects for a massive scalar field, induced by two planar boundaries in background of a linearly expanding spatially flat FRW spacetime for an arbitrary number of spatial dimensions are investigated for general curvature coupling parameter and for Robin boundary conditions on the boundaries. The adiabatic and conformal vacua are specified. The VEVs of the field squared and of the energy-momentum tensor are investigated for a massive conformally coupled field. The influence of the gravitational field on the local characteristics of the vacuum state is essential at distances from the boundaries larger than the curvature radius of the background spacetime. In contrast to the Minkowskian bulk, at large distances the boundary-induced VEVs follow as power law for both massless and massive fields. The Casimir forces acting on the separate plates do not coincide if the corresponding Robin coefficients are different.

4. The Hadamard function and the VEVs of the field squared and energy-momentum tensor are evaluated for a massless conformally coupled scalar field in dS spacetime with general number of spatial dimensions and foliated by spatial sections of negative constant curvature, assuming that the field is prepared in the H-vacuum state. The BD vacuum state is interpreted as thermal with respect to the H-vacuum. The relations obtained for the difference in the VEVs for the Bunch-Davies and H-vacua are compared with the corresponding relations for the Fulling-Rindler and Minkowski vacua in flat spacetime.

5. Complete set of modes and the Hadamard function are constructed for a scalar field inside and outside a sphere in dS spacetime with general number of spatial dimensions and foliated by negative constant curvature spaces, assuming that the field obeys Robin boundary condition on the sphere. The contributions in the Hadamard function induced by the sphere are explicitly separated and the VEVs of the field squared and energy-momentum tensor are investigated for the H-vacuum. In the flat spacetime limit the latter is reduced to the conformal vacuum in the Milne universe and is different from the maximally symmetric BD vacuum state. The vacuum energy-momentum tensor has a nonzero off-diagonal component that describes an energy flux in the radial direction. Depending on the constant in Robin boundary condition and also on the radial coordinate, the energy flux can be directed either from the sphere or towards the sphere. The influence of the gravitational field is essential at late stages of the expansion.

6. FC is investigated in dS spacetime with general number of spatial dimensions by using the cutoff function regularization. In order to fix the renormalization ambiguity for massive fields an additional condition is imposed, requiring the condensate to vanish in the infinite mass limit. For a massless field the FC vanishes for odd number of spatial dimensions and is nonzero for their even number. Depending on the number of spatial dimensions the fermionic condensate can be either positive or negative. The change in the sign of the condensate may lead to instabilities in interacting field theories.

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ԱՄՓՈՓԱԳԻՐ

Ատենախոսությունում հետազոտված են տարածաժամանակի կորության և սահմանների համակցված ազդեցությունը քվանտային վակուումի հատկությունների վրա՝ սկալյար և ֆերմիոնային դաշտերի համար: Որպես ֆոնային երկրաչափություններ դիտարկված են Միլնի տիեզերքը, գծային ընդարձակվող Ֆրիդման-Ռոբերտսոն-Ուոլքերի տարածաժամանակը և բացասական հաստատուն կորությամբ տարածություններով շերտավորված դե Սիտտերի տարածաժամանակը:

1. Ուսումնասիրված են դաշտի քառակուսու և էներգիա-իմպուլսի թենզորի վակուումային միջինները զրոյական զանգվածով սկալյար դաշտի համար տարածական չափողականությունների կամայական թվով Միլնի տիեզերքում՝ ենթադրությամբ, որ դաշտը գտնվում է C-վակուումում: C- և Մինկովսկու վակուումներին համապատասխանող Վայթմանի ֆունկցիաների տարբերության համար ստացվել է ինտեգրալային ներկայացում: Մինկովսկու վակուումային վիճակը կարելի է մեկնաբանել որպես ջերմային վիճակ կոնֆորմ վակուումի համեմատ: Ջերմային ֆակտորը Բոզե-Այնշտայնի տիպի է կենտ թվով և Ֆերմի-Դիրակի տիպի՝ զույգ թվով տարածական չափողականություններում:

2. Հետազոտված է սֆերիկ սահմանի ազդեցությունը զանգվածեղ սկալյար դաշտի վակուումային ֆլուկտուացիաների վրա կամայական թվով տարածական չափողականություններով Միլնի տիեզերքում, ենթադրելով, որ սֆերայի վրա դաշտը բավարարում է Ռոբինի եզրային պայմանին: C-վակուումի դեպքում Հադամարի ֆունկցիան տրոհվել է առանց սահմանի և սֆերայով մակածված ներդրումների: Վերջինիս համար ստացվել է ինտեգրալային ներկայացում սֆերայի ներքին և արտաքին տիրույթներում: Որպես վակուումային վիճակի կարևոր լոկալ բնութագրեր հետազոտվել են դաշտի քառակուսու և էներգիա-իմպուլսի թենզորի միջինները: Վակուումային էներգիա-իմպուլսի թենզորն ունի ոչ անկյունագծային բաղադրիչ, որը համապատասխանում է շառավղային ուղղությամբ էներգիայի հոսքի:

3. Հետազոտված են երկու հարթ սահմաններով մակածված քվանտային վակուումային երևույթները զանգվածեղ սկալյար դաշտի համար գծային ընդարձակվող տարածականորեն հարթ Ֆրիդման-Ռոբերտսոն-Ուոլքերի մոդելում, կորության հետ կապի պարամետրի ընդհանուր դեպքում և սահմանների վրա Ռոբինի եզրային պայմանների համար: Բնորոշված են ադիաբատ և կոնֆորմ վակուումները: Հետազոտված են դաշտի քառակուսու և էներգիա-իմպուլսի թենզորի վակուումային միջինները կոնֆորմ կապված զանգվածեղ դաշտի համար: Գրավիտացիոն դաշտի ազդեցությունը վակուումային վիճակի լոկալ բնութագրերի վրա էական է սահմաններից այնպիսի հեռավորությունների վրա, որոնք ավելի մեծ են քան ֆոնային տարածաժամանակի կորության շառավիղը: Ի տարբերություն Մինկովսկու տարածաժամանակի, մեծ հեռավորությունների վրա սահմաններով մակածված վակուումային միջինները նվազում են աստիճանային օրենքով ինչպես զրոյական զանգվածով, այնպես էլ զանգվածեղ դաշտերի համար: Առանձին թիթեղների վրա ազդող Կազիմիրի ուժերը չեն համընկնում, երբ համապատասխան Ռոբինի գործակիցները տարբեր են:

4. Հաշվարկված են Հադամարի ֆունկցիան և դաշտի քառակուսու ու էներգիա-իմպուլսի թենզորի վակուումային միջինները զրոյական զանգվածով կոնֆորմ

կապված սկալյար դաշտի համար տարածական չափողականությունների կամայական թվով և բացասական հաստատուն կորությամբ տարածական շերտերով շերտավորված dS տարածաժամանակում, ենթադրելով որ դաշտը գտնվում է H-վակուումային վիճակում: Բանչ-Դևիսի վիճակը մեկնաբանվում է որպես ջերմային՝ H-վակուումի նկատմամբ: Բանչ-Դևիսի և H-վակուումների վակուումային միջինների տարբերության համար ստացված առնչությունները համեմատված են հարթ տարածաժամանակում Ֆուլլինգ-Ռինդլերի և Մինկովսկու վակուումների միջև համապատասխան առնչությունների հետ:

5. Կառուցված են մոդաների լրիվ դասը և Հադամարի ֆունկցիան սկալյար դաշտի համար սֆերայի ներսում և դրսում բացասական հաստատուն կորությամբ տարածություններով շերտավորված dS տարածաժամանակում, ենթադրելով որ դաշտը բավարարում է Ռոբինի եզրային պայմանին սֆերայի վրա: Հադամարի ֆունկցիայում բացահայտ կերպով առանձնացված են սֆերայով մակածված ներդրումները և հետազոտված են դաշտի քառակուսու և էներգիա-իմպուլսի թենզորի վակուումային միջինները H-վակուումի համար: Հարթ տարածաժամանակի սահմանում վերջինս հանգում է Միլնի տիեզերքում կոնֆորմ վակուումին և տարբերվում է մաքսիմալ սիմետրիկ Բանչ-Դևիսի վակուումային վիճակից: Վակուումային էներգիա-իմպուլսի թենզորն ունի զրոյից տարբեր ոչ անկյունագծային բաղադրիչ, որը նկարագրում է շառավղային ուղղությամբ էներգիայի հոսքը: Կախված Ռոբինի եզրային պայմանի հաստատունից և շառավղային կոորդինատից, էներգիայի հոսքը կարող է ուղղված լինել ինչպես սֆերայից դեպի դուրս, այնպես էլ դեպի սֆերան: Գրավիտացիոն դաշտի ազդեցությունն էական է ընդարձակման ուշ փուլերում:

6. Հետազոտված է ֆերմիոնային կոնդենսատը տարածական չափողականությունների կամայական թվով dS տարածաժամանակում, օգտագործելով կտրող ֆունկցիայով ռեգուլյարիզացիան: Զանգվածեղ դաշտերի համար վերանորմավորման անորոշությունը բացառելու նպատակով լրացուցիչ պայման է դրվում, պահանջելով որ կոնդենսատը զրոյանա անվերջ զանգվածի սահմանում: Զրոյական զանգվածով դաշտի համար ֆերմիոնային կոնդենսատը զրոյանում է կենտ թվով տարածական չափողականությունների դեպքում և զրոյից տարբեր է զույգ չափողականություններում: Տարածական չափողականությունների թվից կախված ֆերմիոնային կոնդենսատը կարող է լինել ինչպես դրական, այնպես էլ բացասական: Փոխազդող դաշտերի տեսություններում կոնդենսատի նշանի փոփոխությունը կարող է հանգեցնել անկայունությունների:

ПЕТРОСЯН ТИГРАН
НЕКОТОРЫЕ КВАНТОВЫЕ ЭФФЕКТЫ В КОСМОЛОГИЧЕСКИХ МОДЕЛЯХ

В диссертации исследованы комбинированные эффекты кривизны пространства-времени и границ на свойства квантового вакуума для скалярного и фермионного полей. В качестве фоновых геометрий рассмотрены вселенная Милна, линейно расширяющееся пространство-время Фридмана-Робертсона-Уолкера и пространство-время де Ситтера с пространственным расслоением отрицательной постоянной кривизны.

1. Исследованы вакуумные средние квадрата поля и тензора энергии-импульса для безмассового скалярного поля во вселенной Милна с произвольным числом пространственных измерений в предположении, что поле находится в C-вакууме. Получено интегральное представление для разности функций Вайтмана, соответствующих C-вакууму и вакууму Минковского. Вакуумное состояние Минковского можно интерпретировать как тепловое по отношению к конформному вакууму. Тепловой фактор является типа Бозе-Эйнштейна в пространстве нечетной и типа Ферми-Дирака для четной размерности пространственных измерений.

2. Исследовано влияние сферической границы на вакуумные флуктуации массивного скалярного поля на фоне вселенной Милна с произвольным числом пространственных измерений, предполагая, что на сфере поле удовлетворяет граничному условию Робина. Функция Адамара для C-вакуума разложена на вклады, обусловленный отсутствием границ и индуцированный сферой. Для последнего получено интегральное представление во внутренней и во внешней областях. В качестве важных локальных характеристик вакуумного состояния исследованы вакуумные средние квадрата поля и тензора энергии-импульса. Вакуумный тензор энергии-импульса имеет недиагональную составляющую, соответствующую потоку энергии вдоль радиального направления.

3. Исследованы квантовые вакуумные эффекты для массивного скалярного поля, индуцированные двумя плоскими границами на фоне линейно расширяющегося пространственно плоского пространства-времени Фридмана-Робертсона-Уолкера для произвольного числа пространственных измерений, при произвольном значении параметра связи с кривизной и для граничных условий Робина на границах. Определены адиабатический и конформный вакуумы. Исследованы вакуумные средние квадрата поля и тензора энергии-импульса для массивного конформно связанного поля. Влияние гравитационного поля на локальные характеристики вакуумного состояния существенно на расстояниях от границ, превышающих радиус кривизны фонового пространства-времени. В отличие от пространства-времени Минковского, на больших расстояниях вакуумные средние, индуцированные границами, меняются как степенные функции как для безмассовых, так и для массивных полей. Силы Казимира, действующие на отдельные пластины, не совпадают, когда соответствующие коэффициенты Робина различны.

4. Вычислены функция Адамара и вакуумные средние квадрата поля и тензора энергии-импульса для конформно связанного безмассового скалярного поля в пространстве-времени dS с произвольным числом пространственных измерений

и с пространственным расслоением отрицательной постоянной кривизны, в предположении, что поле находится в H-вакуумном состоянии. Состояние Банча-Дэвиса интерпретируется как тепловое по отношению к H-вакууму. Соотношения, полученные для разности вакуумных средних вакуума Банча-Дэвиса и H-вакуума, сравниваются с соответствующими соотношениями для вакуумов Фуллинга-Риндлера и Минковского в плоском пространстве-времени.

5. Построены полный набор мод и функция Адамара для скалярного поля внутри и вне сферы в пространстве-времени dS с произвольным числом пространственных измерений и расслоенным на пространства с отрицательной постоянной кривизной в предположении, что поле удовлетворяет граничному условию Робина на сфере. Выделены в явном виде вклады в функцию Адамара, индуцированные сферой, и исследованы вакуумные средние квадрата поля и тензора энергии-импульса для H-вакуума. В пределе плоского пространства-времени последний сводится к конформному вакууму во вселенной Милна и отличается от максимально симметричного состояния вакуума Банча-Дэвиса. Вакуумный тензор энергии-импульса имеет ненулевую недиагональную компоненту, которая описывает поток энергии в радиальном направлении. В зависимости от константы в граничном условии Робина, а также от радиальной координаты, поток энергии может быть направлен как от сферы, так и к сфере. Влияние гравитационного поля существенно на поздних стадиях расширения.

6. Исследован фермионный конденсат в пространстве-времени dS с произвольным числом пространственных измерений с использованием регуляризации функцией обрезания. Чтобы исключить неоднозначность перенормировки для массивных полей, накладывается дополнительное условие, требующее обращения конденсата в нуль в пределе бесконечной массы. Для безмассового поля фермионный конденсат обнуляется при нечетном числе пространственных измерений и отличен от нуля при их четных значениях. В зависимости от числа пространственных измерений фермионный конденсат может быть как положительным, так и отрицательным. В теориях взаимодействующих полей изменение знака конденсата может привести к неустойчивостям.