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Կարապետյան Մելիք Մուշեղի

ՓՈԽԱԶԴՈՂ ԲԱՐՁՐ ՍՊԻՆՆԵՐՈՎ ՏԵՍՈՒԹՅՈՒՆՆԵՐ ՀԱՐԹ ԵՎ ԱՆՏԻ ԴԵ ՍԻՏԵՐԻ ՏԱՐԱԾՈՒԹՅՈՒՆՆԵՐՈՒՄ

Ա.04.02 - «Տեսական ֆիզիկա» մասնագիտությամբ ֆիզիկամաթեմատիկական գիտությունների թեկնածուի գիտական աստիճանի հայցման ատենախոսության

ՍԵՂՄԱԳԻՐ

ԵՐԵՎԱՆ - 2022

A. I. ALIKHANYAN NATIONAL SCIENCE LABORATORY
(YEREVAN PHYSICS INSTITUTE)

Melik Karapetyan

INTERACTING HIGHER SPIN THEORIES IN FLAT AND ADS SPACES

SYNOPSIS

of Dissertation in 01.04.02 - Theoretical Physics presented for the degree of candidate in physical and mathematical sciences

YEREVAN - 2022

Ատենախոսության թեման հաստատվել է Ա. Ի. Ալիխանյանի անվան Ազգային Գիտական Լաբորատորիայի (ԵրՖԻ) գիտական խորհրդում:

Գիտական ղեկավար՝
Ֆիզմաթ. գիտ. դոկտոր Ռուբեն Պետրոսի Մանվելյան (ԱԱԳԼ)

Պաշտոնական ընդդիմախոսներ՝
Ֆիզմաթ. գիտ. դոկտոր Արմեն Պետրոսի Ներսեսյան (ԱԱԳԼ)
Ֆիզմաթ. գիտ. դոկտոր Ալեքսեյ Պետրովիչ Իսաև (ՄՀՄԻ)

Առաջատար կազմակերպություն՝
Թբիլիսիի Պետական Համալսարանի Ռազմաձեի մաթեմատիկական ինստիտուտ, տեսական ֆիզիկայի բաժին

Ատենախոսության պաշտպանությունը կայանալու է 2022 թ. սեպտեմբերի 8-ին ժամը 14:00-ին, ԱԱԳԼ-ում գործող ԲՈԿ-ի 024 «Ֆիզիկայի» մասնագիտական խորհրդում (Երևան, 0036, Ալիխանյան եղբայրների փ. 2):
Ատենախոսությանը կարելի է ծանոթանալ ԱԱԳԼ-ի գրադարանում:
Սեղմագիրը առաքված է 2022 թ. Հուլիս 30-ին:

Մասնագիտական խորհրդի գիտական քարտուղար՝
Ֆիզմաթ. գիտ. դոկտոր Հրաչյա Հովհաննեսի Մարուքյան

The subject of the dissertation is approved by the scientific council of the A. I. Alikhanyan National Science Laboratory (YerPhI)

Scientific supervisor:
Doctor of ph-math. sciences Ruben Manvelyan (AANL)

Official opponents:
Doctor of ph-math. sciences Armen Nersesyan (AANL)
Doctor of ph-math. sciences Alexei Isaev (JINR)

Leading Organization:
Department of Theoretical Physics, Razmadze Mathematical Institute, Tbilisi State University.

The defense will take place on the 8^{th} of September 2022 at 14:00 during the "Physics" professional council's session of SCC 024 acting within AANL (2 Alikhanyan Brothers str., 0036, Yerevan)

The dissertation is available at AANL library.
The synopsis is sent out on the 30^{th} of July, 2022.

Scientific secretary of the special council
Doctor of ph-math. sciences Hrachya Marukyan

Abstract

This thesis is devoted to the investigation of the interacting higher spin gauge theories in flat and AdS spaces. First of all we start with the introduction to the Fronsdal formalism for Higher spin fields. We calculate first the equations of motion for lower spin 1,2,3 cases and then turn to it's arbitrary spin s generalization. We show that *in De Donder gauge the equation of motion becomes harmonic*. Then to simplify our work with HS objects we introduce polynomial notation, which is widely used in HS theories and leads to working with polynomials instead of symmetric tensors with many indices. Next, we present the Fronsdal Lagrangian and equation of motion using the polynomial notation. After that we introduce the Neother's procedure, which is standard method for perturbative derivation of interaction using deformations of free equations and symmetries in self consistent way.

After this, we concentrate on the computation of cubic interaction for a higher spin on AdS space in explicit covariant form. We succeeded in solving all necessary *recurrence relations* to finalize full radial pullback of the main term of cubic self-interaction for higher spin gauge fields in Fronsdal's formulation from flat to one dimension less AdS_{d+1} space. As a result, non-trivial solutions of recurrence relations lead to the possibility to obtain the complete set of AdS_{d+1} dimensional interacting terms with all curvature corrections, including trace and divergence terms from any interaction term in d+2 dimensional flat space.

Next, we solved the non-trivial task of construction of interacting Lagrangian for the higher spin field in physical gauge, using the full power of Noether's procedure. As a result, the linear on-field gauge transformation is obtained, and the corresponding commutator of transformation is analyzed. To understand the closure of this algebra, the right-hand side of this commutator is classified with respect to gauge transformations coming from cubic interactions with different higher spin symmetric tensor fields and mixed symmetry tensor fields transformations.

During the all research process, we have intensively used the Wolfram Mathematica along with *xAct*, *xTensor*, *xTras* packages to model complex problems and solve them programmatically. We have developed methods and tools for working with higher spin fields in Wolfram Mathematica, and these methods are generic enough to be used on other use-cases as well. We introduce some of the modeling approaches in Wolfram Mathematica language and include the developed methods and functions as codding snippets in the separate section on each chapter.

Relevance and motivation

Higher spin gauge theories are an essential part of modern theoretical physics. The spectrum of these theories includes graviton as a massless spin-two field. The similar mass-less spin two modes exists in the (super)String theory also, but there all remaining higher spin hierarchy is essentially massive. So, therefore, massless higher spin gauge theory with infinite number of massless higher spin modes can be considered as a theory with even higher than in the string theory case gauge symmetries and interpret latter as a spontaneously broken phase of HS gauge theory. Moreover, these theories are supposed to be consistent quantum theories, hence they all include gravity as a part. Higher Spin Theories are interesting polygon for checking the AdS/CFT correspondence, where there are many conjectures related weakly coupled higher spin theories living in the bulk to strong coupled conformal field theories on the boundary of Anti de Sitter space. There are a lot of interesting problems for research, such as the construction of interacting Higher Spin theories with corresponding equations of motions and interacting Lagrangians.

Construction of an interacting Higher Spin (HS) gauge theory is a problem with some permanent background interest for more than the last thirty years, starting from early work [7]. Periodically, one can observe a growing interest in this object of investigation, mainly realized as some success in constructing cubic interaction in AdS or flat background and in connection with AdS/CFT and HS gravity in various dimensions. These attempts were always attractive as one more way to relate quantum theory with General Relativity and investigate HS gauge fields on the same shelf with gravity or understand the uniqueness of gravity (spin 2 field) compared to other members of the HS hierarchy. It is worth recalling that even though consistent equations of motion [8] for *interacting higher spin* fields have been known for many years, the action principle for these theories remains problematic. Construction of HS interaction Lagrangian in itself is a very interesting task due to its complexity and necessity to develop non-trivial computing techniques for even small achievements. During the previous 10-12 years, there was significant progress in this area, especially in the understanding of the construction and structure of cubic interaction in different approaches, dimensions, and backgrounds. Yet, our knowledge is far from being complete and seems to be bounded to the idea that quartic interaction should be non-local.

Aim of the dissertation

The main goal of this thesis is to analyze the Higher spin interactions in flat and AdS spaces. In this work, we focused on two problems. One of which was the construction of cubic interaction in AdS_{d+1} space in explicit covariant form.

- Construction of the main term of the cubic interaction in flat space.
- The structure of the polynomial coefficients and the solution of the recurrence relations.
- Problem modeling in Wolfram Mathematica
- Pullback of the main term of the cubic self-interaction

The second problem was the construction of the some special case of local quartic interaction for the higher spin field in physical gauge.

- Computing the commutator of δ_1 transformations for spin four
- Expressing the commutator and variations using generalized Christopher symbols
- Classification of the commutator and interpretation of the right-hand side.

Results submitted for defense

- We were able to successfully finalize the radial pullback procedure for the main term of cubic self-interaction by solving all necessary recurrence relations. The solutions of these equations lead to the possibility of obtaining the full set of AdS_{d+1} dimensional interacting terms with all curvature corrections, including trace and divergence terms from any interaction term in $d+2$ dimensional flat space.
- Using Wolfram Mathematica programming language, we developed general methods for solving recurrence relations which we faced during the radial pullback procedure. These methods are also generic enough to be used for solving different recurrence relations in other problems.
- We have considered local quartic interaction between higher-spin gauge field and scalar field. Using the full power of Neother's procedure, we successfully constructed interacting Lagrangian for this special case in a physical gauge.
- We successfully constructed and analyzed the commutator of the linearized higher spin gauge field transformation and successfully classified the right-hand side of the commutator. As an important result, we discovered that on the right-hand side of the commutator, there are terms with mixed symmetry.

- Using Wolfram Mathematica programming language, we have developed generic functions and methods which introduced the possibility of working with symmetric tensors with higher rank using the polynomial notation, which significantly makes computation much easier and human-readable.

Novelty of the works

The usual methods of constructing an interacting Lagrangian in a perturbative way suppose to use the Fronsdal metric formalism for free fields [9]. The essential point here is that during perturbative (Noether method) construction of interaction for HS models, one can see and construct corresponding perturbative deformation of the free field's gauge transformations and then meet the certain difficulties connected with the locality of the theory beyond the cubic order [10–14]. While, the cubic interaction is the main building object of HS interaction, not all problems are solved in a fast way, even on a cubic level. Despite the fact that cubic interaction in AdS space has formulation developed in ambient space some years ago [15–19] the direct formulation on the language of AdS_{d+1} covariant derivatives was still unknown and realized before in [19] for some simplest part of interaction only. From other side realization of the Noether program directly in AdS space [20] is also extremely difficult due to noncommutativity of covariant derivatives in space with constant curvature. Therefore at the moment, the only way to see this interaction in AdS space directly is to continue the approach defined in [19], which we did in [3] and in this thesis.

Even though during the last ten-twelve years we observed significant progress in the understanding of construction and structure of cubic interaction in different approaches, dimensions and backgrounds [7]- [21], our knowledge about higher-order vertices is far from complete and seems to be bounded by the idea that the quartic one should be non-local in general. We were able to investigate the algebra of gauge transformations by considering the commutator of gauge transformations of fourth-rank higher spin fields, expanded, and restricted, to the linear over gauge fields approximation and were able to successfully classify the r.h.s of commutator. Despite the restricted scope of the problem, it is technically very complicated due to the usual for higher spin theories abundance of indexes, with different symmetries imposed. Especially for handling this and similar problems, we adapted "Wolfram Mathematica" packages and showed that they allow one to effectively handle this problem. As a result, the calculation of the commutator of linear gauge transformations on the fourth rank tensor field is obtained. The result is completely classified, particularly in the r.h.s. are separated all known gauge transformations. Besides them, there are terms, which probably can be interpreted as gauge symmetries of fifth and sixth rank tensor fields gauge transformations, and remaining are the number of additional terms. For latter's it is shown that they all should belong to the (yet unknown) mixed symmetry gauge transformations. Altogether, the results are are step forward in the understanding of gauge symmetries algebra in the higher spin gauge theories.

Practical value

The result of this thesis can be used for the development of new approaches in the important and contemporary area of theoretical physics such as investigations of quantum gravity, dark matter, string theory, and any other modification of previously known theories with an aim to go beyond standard knowledge. The results, specifically the modeling part in Wolfram Mathematica is very generic and can be used for solving other problems in Higher Spin Gauge Theory area.

Length and structure of the dissertation

The dissertation contains 3 chapters, summary and the bibliography. The first chapter is the introduction the other two chapters describe our findings. In the summary we sum up all our results.

Content

CHAPTER 1

The first chapter of the thesis is an introduction where we set up some general concepts that we use in the next chapters. We present the Fronsdal formalism: i.e. Fronsdal Lagrangian and equation of motion supplemented with corresponding linearized gauge symmetry. Also, we introduce the polynomial notation for working with higher spin objects which simplifies the job of working with tensorial objects with a lot of indices. Then we introduce Neother's procedure which is widely used as a perturbative method for computing the interactions.

CHAPTER 2

The second chapter is devoted to the cubic interaction of the higher spin gauge theory in AdS_{d+1} space.It is based on the following papers [1,3,4]. Cubic interaction in AdS space has formulation developed in ambient space some years ago [15]- [19] the direct formulation on the language of AdS_{d+1} covariant derivatives was still unknown and realized before in [19] for some simplest part of interaction only. On the other side, the realization of the Noether program directly in AdS space [20] is also extremely difficult due to the noncommutativity of covariant derivatives in space with constant curvature. Therefore at the moment, the only way to see this interaction in AdS space directly is to continue the approach defined in [19].

So the main purpose of this chapter is to complete the first part of the program defined in [19] where authors considered a version of the radial reduction (or pullback) formalism to obtain a cubic interaction of higher spin gauge fields in AdS_{d+1} space from the corresponding cubic interaction in a flat $d+2$ dimensional background. The crucial point in [19] was to write AdS_{d+1} cubic interaction terms directly in $d+1$ dimensional space using AdS_{d+1} covariant derivatives. This was done there only for the main term and AdS_{d+1} curvature corrections without trace terms. The result was enough elegant but expressed only one simplest type of correction terms. Here we complete the setup proposed in appendixes of [19] for all correction terms coming from main (in other words, transverse and traceless) terms in flat space. The key point of this chapter is that we succeeded in formulation and solution of the corresponding recurrence relations to complete radial pullback from $d+2$ dimensional flat ambient space to AdS_{d+1} in all orders of curvature expansion including all possible trace terms. Another important point of this consideration is that we constructed a general pullback prescription for objects with higher derivatives of higher spin gauge fields to realize the corresponding reduction for all other terms of cubic interaction pushing this important remaining task of our program in the field of just technical work which can be done in the future without additional difficulties. This we are left for future publication.

In the first section, we presented and applied the correct radial pullback procedure for the free field reconciled with gauge invariance. Our formulation slightly differs from approaches used in [15]- [19] but is completely equivalent to them and more suitable for application to cubic interaction. In the second section, we considered pullback for the high power of flat derivatives of HS field in $d+2$ dimensional space to power of covariant derivatives in AdS_{d+1} which is the most important ingredient of cubic interaction.

We formulate the main term of cubic interaction in the following way

$$\mathcal{L}_I^{main}(h^{(s_1)}(X,a^A),h^{(s_2)}(X,b^A),h^{(s_3)}(X,c^A)) =$$
$$\sum_{Q_{ij}} C^{s_1,s_2,s_3}_{Q_{12},Q_{23},Q_{31}} \int d^{d+2}X *_{c^A}^{Q_{31}+n_3} K^{(s_1)}(Q_{31},n_3;c^A,a^A;X)$$
$$*_{a^A}^{Q_{12}+n_1} K^{(s_2)}(Q_{12},n_1;a^A,b^A;X) *_{b^A}^{Q_{23}+n_2} K^{(s_3)}(Q_{23},n_2;b^A,c^A;X), \tag{1}$$

where

$$K^{(s_1)}(Q_{12},n_1;a^A,b^A;X) = (a^A\partial_{b^A})^{Q_{12}}(a^B\partial_B)^{n_1}h^{(s_1)}(X;b^C). \tag{2}$$

The most important advantage of this form is that here we can express our cubic interaction as a cube of the above bitensor function with cyclic index contraction.

During the development of the radial pullback scheme for the main object of cubic interaction, several recurrence relations pop up from the non-commutative algebra, which we successfully solved and completed the pullback of the main term of cubic interaction with all *AdS* corrections supplemented by corresponding trace terms and got the following result for the interaction:

$$\mathcal{L}_I^{main} = \int du e^{(d+2s-4)u} d^{d+1}x\sqrt{g} \sum_{N\geq 0}\sum_{K\geq 0} \frac{(-1)^N s!}{(s-2(N+K))!}$$
$$\sum_{\substack{\tilde{\alpha},\tilde{\beta},\tilde{\gamma} \\ \tilde{\alpha}+\tilde{\beta}+\tilde{\gamma}=s-2(N+K)}} \binom{s-2(N+K)}{\tilde{\alpha},\tilde{\beta},\tilde{\gamma}}$$
$$\sum_{\substack{\{n_i\}_{i=1,2,3} \\ \sum n_i=N}} \sum_{\substack{\{P_i,K_i\}_{i=1,2,3} \\ P_i=n_i+n_{i+1}+2K_i \\ \sum K_i=K}} \sum_{\substack{\{p_i,k_i,\tilde{p_i},\tilde{k_i}\}_{i=1,2,3} \\ p_i+\tilde{p_i}=P_i; k_i+\tilde{k_i}=K_i}}$$
$$\frac{*_{a^\mu}^{\tilde{\gamma}+\tilde{\alpha}+N+2(K_3+K_1)} *_{b^\mu}^{\tilde{\alpha}+\tilde{\beta}+N+2(K_1+K_2)} *_{c^\mu}^{\tilde{\beta}+\tilde{\gamma}+N+2(K_2+K_3)}}{\binom{\tilde{\gamma}+\tilde{\alpha}+N+2(K_3+K_1)+n_1}{n_1}\binom{\tilde{\alpha}+\tilde{\beta}+N+2(K_1+K_2)+n_2}{n_2}\binom{\tilde{\beta}+\tilde{\gamma}+N+2(K_2+K_3)+n_3}{n_3}}$$
$$(a,\partial_b)^{\tilde{\gamma}+N+2K_3}(a,\nabla)^{\tilde{\alpha}}\,\Xi^{2K_1}[\tilde{\gamma},\tilde{\alpha},n_2,p_1,k_1,\tilde{p_1},\tilde{k_1},a^2,H_1]h^{(s)}(b^\mu)$$
$$(b,\partial_c)^{\tilde{\alpha}+N+2K_1}(b,\nabla)^{\tilde{\beta}}\,\Xi^{2K_2}[\tilde{\alpha},\tilde{\beta},n_3,p_2,k_2,\tilde{p_2},\tilde{k_2},b^2,H_2]h^{(s)}(c^\mu)$$
$$(c,\partial_a)^{\tilde{\beta}+N+2K_2}(c,\nabla)^{\tilde{\gamma}}\,\Xi^{2K_3}[\tilde{\beta},\tilde{\gamma},n_1,p_3,k_3,\tilde{p_3},\tilde{k_3},c^2,H_3]h^{(s)}(a^\mu) \tag{3}$$

where

$$\tilde{\Theta}[\gamma,\alpha,n_2,p_1,k_1,\tilde{p_1},\tilde{k_1},a^2,H_1] = \tag{4}$$
$$\frac{\gamma!}{\tilde{\alpha}!}\,\Xi^{2K_1}[\tilde{\gamma},\tilde{\alpha},n_2,P_3,p_1,k_1,\tilde{p_1},\tilde{k_1},a^2,H_1]$$
$$\tilde{\Theta}[\alpha,\beta,n_3,p_2,k_2,\tilde{p_2},\tilde{k_2},b^2,H_2] = \tag{5}$$
$$\frac{\alpha!}{\tilde{\beta}!}\,\Xi^{2K_2}[\tilde{\alpha},\tilde{\beta},n_3,P_1,p_2,k_2,\tilde{p_2},\tilde{k_2},b^2,H_2]$$
$$\tilde{\Theta}[\beta,\gamma,n_1,p_3,k_3,\tilde{p_3},\tilde{k_3},c^2,H_3] = \tag{6}$$
$$\frac{\beta!}{\tilde{\gamma}!}\,\Xi^{2K_3}[\tilde{\beta},\tilde{\gamma},n_1,P_2,p_3,k_3,\tilde{p_3},\tilde{k_3},c^2,H_3]$$

and

$$\Xi^{2K_1}[\tilde{\gamma},\tilde{\alpha},n_2,P_3,p_1,k_1,\tilde{p_1},\tilde{k_1},a^2,H_1]$$
$$= \frac{(\tilde{\alpha}+\tilde{p_1})!(a^2)^{k_1}}{(\tilde{\gamma}+P_3-\tilde{p_1}+2\tilde{k_1}+n_2)!}\binom{\tilde{p_1}-2\tilde{k_1}}{n_2}$$
$$\xi_{k_1}^{p_1+1}(\tilde{\alpha}+\tilde{p_1})W^{\tilde{k_1}}(a^2,H_1)\,, \tag{7}$$
$$\Xi^{2K_2}[\tilde{\alpha},\tilde{\beta},n_3,P_1,p_2,k_2,\tilde{p_2},\tilde{k_2},b^2,H_2]$$
$$= \frac{(\tilde{\beta}+\tilde{p_2})!(a^2)^{k_2}}{(\tilde{\alpha}+P_1-\tilde{p_2}+2\tilde{k_2}+n_3)!}\binom{\tilde{p_2}-2\tilde{k_2}}{n_3}$$
$$\xi_{k_2}^{p_2+1}(\tilde{\beta}+\tilde{p_2})W^{\tilde{k_2}}(b^2,H_2)\,, \tag{8}$$
$$\Xi^{2K_3}[\tilde{\beta},\tilde{\gamma},n_1,P_2,p_3,k_3,\tilde{p_3},\tilde{k_3},c^2,H_3]$$
$$= \frac{(\tilde{\gamma}+\tilde{p_3})!(a^2)^{k_3}}{(\tilde{\beta}+P_2-\tilde{p_3}+2\tilde{k_3}+n_1)!}\binom{\tilde{p_3}-2\tilde{k_3}}{n_1}$$
$$\xi_{k_3}^{p_3+1}(\tilde{\gamma}+\tilde{p_3})W^{\tilde{k_3}}(c^2,H_3)\,. \tag{9}$$

As a result we constructed all *AdS* corrections including trace and divergence terms to the main term of the cubic self-interaction by a slightly modified method of radial pullback (reduction) proposed in [19] where all quantum fields are carried by a real AdS space and corresponding interaction terms expressed through the covariant *AdS* derivatives. For given spin s and $\Delta_{min} = s$ we derived all curvature correction terms (3) in the form of series of terms with numbers $s - 2(N+K)$ of derivatives, where $0 \leq N+K \leq \frac{s}{2}$. The latter is the number of seized pair of derivatives replaced by corresponding power of $1/L^2$ and K is the sum of power of a^2, b^2, c^2 terms connected with trace and divergent correction terms produced from the main term of interaction after pullback. Correction terms appear with coefficients that are polynomials in the dimension $d+1$ and spin number s with rational coefficients. Now we can expect that the same method can be used for the derivation of the AdS corrections to traces and deDonder terms connected with the main term by Noether's procedure derived for the flat case in [22, 23] .

CHAPTER 3

In the third chapter, we construct some special local quartic interactions of two scalars and two spin four fields using standard Noether's procedure. The chapter is based on the following papers [2, 5, 6]. The interesting points of this special construction are the following:

- First we see that to close Noether's procedure we should add additional cubic interaction of scalar with other spin gauge fields and corresponding HS gauge symmetries.
- Second important point that we constructing quartic vertex we derive fixed linear in gauge field gauge transformation of our HS field $\delta_1^{(\epsilon)}$ and then be able to investigate the closure of commutators of two such a transformation

  $$[\delta_1^{(\eta)}\delta_1^{(\epsilon)}] \sim \delta_1^{([\eta,\epsilon])} + \text{additional terms}$$

  and understand whether it leads to non-locality or not.

To construct the quartic interaction we start from the cubic term

$$L_1 \sim h^{\mu\nu\lambda\rho}\partial_\mu\partial_\nu\Phi\partial_\lambda\partial_\rho\Phi, \tag{10}$$

and using known variation:

$$\begin{aligned} \delta_1\Phi &= \varepsilon^{\alpha\beta\gamma}\partial_\alpha\partial_\beta\partial_\gamma\Phi, && (11)\\ \delta_0 h^{\mu\nu\lambda\rho} &= \partial^{(\mu}\varepsilon^{\nu\lambda\rho)}, && (12) \end{aligned}$$

we try to solve the functional equation:

$$\delta_1 L_1(\Phi, h^{(4)}) + \delta_0 L_2(\Phi, h^{(4)}) = 0, \tag{13}$$

and construct unknown quartic interaction and first order gauge variation of spin four field $\delta_1 h^{\mu\nu\lambda\rho}$. Doing that and taking into account that α, β, γ derivatives commute with ε and μ, ν, λ, ρ derivatives commute with h and after long manipulations and multiple partial integrations we arrive to the following expression for the interaction

$$S_2(\Phi, h^{(4)}) = \int d^d x \Big\{ \frac{1}{10} h^{\alpha\beta\gamma}_{\mu} h^{\nu\lambda\rho\mu} \tilde{J}^{(6)}_{\nu\lambda\rho\alpha\beta\gamma}$$
$$-\frac{2}{3} h^{\alpha\beta\gamma}_{\mu} \partial_\alpha \partial_\beta h^{\mu\nu\lambda\rho} J^{(4)}_{\nu\lambda\rho\gamma} + \frac{1}{2} \partial_\nu h^{\alpha\beta\gamma}_{\mu} \partial_\alpha h^{\mu\nu\lambda\rho} J^{(4)}_{\lambda\rho\beta\gamma} - \frac{1}{4} \partial^\alpha h^{\beta\gamma}_{\mu\nu} \partial_\alpha h^{\mu\nu\lambda\rho} J^{(4)}_{\lambda\rho\beta\gamma}$$
$$-\partial^\beta h^{\alpha\gamma}_{\mu\nu} \partial_\alpha h^{\mu\nu\lambda\rho} J^{(4)}_{\lambda\rho\beta\gamma} + \frac{1}{3} \partial^\beta h^{\gamma}_{\mu\nu\lambda} \partial^\alpha h^{\mu\nu\lambda\rho} J^{(4)}_{\rho\alpha\beta\gamma} - \frac{1}{4} h^{\beta\gamma}_{\mu\nu} h^{\lambda\rho\mu\nu} \Box J^{(4)}_{\lambda\rho\beta\gamma} \Big\}, \tag{14}$$

and linear on spin four gauge field transformations fixed by Noether's procedure:

$$\delta_1 h^{\mu\nu\lambda\alpha\beta\gamma}_{(6)} = \varepsilon^{\rho(\alpha\beta} \partial_\rho h^{\gamma\mu\nu\lambda)} + \partial^{(\alpha} \varepsilon^{\beta\gamma}_{\rho} h^{\mu\nu\lambda)\rho}, \tag{15}$$

$$\delta_1 h^{\mu\nu\lambda\rho} = \varepsilon^{\alpha\beta\gamma} \partial_\alpha \partial_\beta \partial_\gamma h^{\mu\nu\lambda\rho} + \partial^{(\mu} \varepsilon^{|\alpha\beta|}_{\gamma} \partial_\alpha \partial_\beta h^{\nu\lambda\rho)\gamma} + \partial^{(\mu} \partial^{\nu} \varepsilon^{|\alpha|}_{\beta\gamma} \partial_\alpha h^{\lambda\rho)\beta\gamma}$$
$$+\partial^{(\mu} \partial^{\nu} \partial^{\lambda} \varepsilon_{\alpha\beta\gamma} h^{\rho)\alpha\beta\gamma}, \tag{16}$$

$$\delta_1 h^{\beta\gamma}_{(2)} = \partial_\mu \partial_\nu \partial_\lambda \partial_\rho \varepsilon^{\alpha\beta\gamma} \partial_\alpha h^{\mu\nu\lambda\rho}. \tag{17}$$

So we prove that Noether's procedure in this particular case can be done for the construction of the local quartic interaction of the scalar and higher spin fields restricted by transverse and traceless gauge conditions.

After some tedious calculations, we arrive to the following result for the commutator of δ_1 gauge transformation.

$$[\delta^{(\omega)}_1, \delta^{(\varepsilon)}_1] h_{\mu\nu\lambda\rho} \sim \left[\varepsilon^{\delta\sigma\eta} \partial_\delta \partial_\sigma \partial_\eta \omega^{\alpha\beta\gamma} + T^{\alpha\beta\gamma}(\partial, \varepsilon, \omega) \right] \Gamma^{(3)}_{\alpha\beta\gamma;\mu\nu\lambda\rho}(h)$$
$$+3\varepsilon^{\delta\sigma\eta} \partial_\delta \partial_\sigma \omega^{\alpha\beta\gamma} R^{(4)}_{\eta\alpha\beta\gamma;\mu\nu\lambda\rho}(h) + \frac{9}{20} \varepsilon^{\sigma\eta}_{\delta} \partial^{[\delta} \omega^{\alpha]\beta\gamma} \partial_{(\sigma} R^{(4)}_{\eta\alpha\beta\gamma);\mu\nu\lambda\rho}(h)$$
$$+[Rem]_{\mu\nu\lambda\rho}(\varepsilon, \omega, h) - (\varepsilon \leftrightarrow \omega), \tag{18}$$

where:

$$\begin{aligned} T^{\alpha\beta\gamma}(\partial, \varepsilon, \omega) &= \frac{1}{4} \partial^{(\alpha} \partial^{\beta} \varepsilon^{\delta\sigma\eta} \delta^{(\omega)}_0 h^{\gamma)}_{\delta\sigma\eta} - \frac{5}{48} \partial^{(\alpha} \varepsilon^{\delta\sigma\eta} \\ &\quad \partial^{\beta} \delta^{(\omega)}_0 h^{\gamma)}_{\delta\sigma\eta} + \frac{7}{16} \partial^{(\alpha} \varepsilon^{\delta\sigma\eta} \partial_\delta \delta^{(\omega)}_0 h^{\beta\gamma)}_{\sigma\eta} \\ &- \frac{1}{16} \partial^{\delta} \varepsilon^{\sigma\eta(\alpha} \partial^{\beta} \delta^{(\omega)}_0 h^{\gamma)}_{\delta\sigma\eta} + \frac{1}{16} \partial^{\delta} \varepsilon^{\sigma\eta(\alpha} \partial_\delta \delta^{(\omega)}_0 h^{\beta\gamma)}_{\sigma\eta}, \end{aligned} \tag{19}$$

and

$$
\begin{aligned}
&[Rem]_{\mu\nu\lambda\rho}(\varepsilon,\omega,h) = \\
&\frac{9}{20}\varepsilon^{\eta\sigma}_{\delta}\partial^{[\delta}\omega^{\alpha]\beta\gamma}\partial_{(\mu}R^{(3)}_{\nu\lambda\rho);\eta\beta\gamma}(H^{(3)}_{[\alpha\sigma]}) + \frac{3}{2}\partial_{(\mu}\varepsilon^{\eta\sigma}_{\delta}\partial^{[\delta}\omega^{\alpha]\beta\gamma}R^{(3)}_{\nu\lambda\rho);\eta\beta\gamma}(H^{(3)}_{[\alpha\sigma]}) \\
&-\frac{9}{40}\varepsilon^{\eta\sigma}_{\delta}\partial^{[\delta}\omega^{\alpha]\beta\gamma}\partial_{(\mu}R^{(3)}_{\nu\lambda\rho);\eta\alpha\gamma}(H^{(3)}_{[\beta\sigma]}) \\
&+\frac{3}{8}\varepsilon^{\eta}_{\sigma\delta}\partial^{[\sigma}\partial^{\left[\delta\right.}\omega^{\left.\alpha\right]\left.\beta\right]\gamma}\partial_{(\mu}\Gamma^{(2)}_{\beta\gamma;\nu\lambda\rho)}(H^{(3)}_{[\eta\alpha]}) + \frac{1}{2}\partial_{(\mu}\varepsilon^{\eta}_{\delta\sigma}\partial^{[\sigma}\partial^{\left[\delta\right.}\omega^{\left.\alpha\right]\left.\beta\right]\gamma}\Gamma^{(2)}_{\beta\gamma;\nu\lambda\rho)}(H^{(3)}_{[\eta\alpha]}) \\
&+\frac{3}{8}\varepsilon^{\sigma\eta}_{\delta}\partial^{[\delta}\omega^{\alpha]\beta\gamma}\partial_{(\mu}\partial_{\nu}\Gamma^{(1)}_{\gamma;\lambda\rho)}(H^{(2)}_{[\eta\alpha][\sigma\beta]}) + \frac{1}{2}\partial_{(\mu}\varepsilon^{\sigma\eta}_{\delta}\partial^{[\delta}\omega^{\alpha]\beta\gamma}\partial_{\nu}\Gamma^{(1)}_{\gamma;\lambda\rho)}(H^{(2)}_{[\eta\alpha][\sigma\beta]}) \\
&+\frac{3}{4}\partial_{(\mu}\partial_{\nu}\varepsilon^{\sigma\eta}_{\delta}\partial^{[\delta}\omega^{\alpha]\beta\gamma}\Gamma^{(1)}_{\gamma;\lambda\rho)}(H^{(2)}_{[\eta\alpha][\sigma\beta]}) \qquad (20)
\end{aligned}
$$

is remaining part of commutator contained transformation described by composed gauge parameter with mixed symmetry of indices in the form of one or two antisymmetrized pairs.

To be more precise when classifying terms on the right side of (18) let us consider each line separately:

1. The first line describes spin four gauge transformation with composite *symmetric rank 3 tensor parameter* in the form

$$
[\omega,\varepsilon]^{\alpha\beta\gamma}\Gamma^{(3)}_{\alpha\beta\gamma;\mu\nu\lambda\rho}(h), \qquad (21)
$$

   where

$$
[\omega,\varepsilon]^{\alpha\beta\gamma} = \varepsilon^{\delta\sigma\eta}\partial_{\delta}\partial_{\sigma}\partial_{\eta}\omega^{\alpha\beta\gamma} + T^{\alpha\beta\gamma}(\partial,\varepsilon,\omega) - (\varepsilon\leftrightarrow\omega). \qquad (22)
$$

2. The second line also corresponds to the transformation of the spin four gauge field in respect to gauge transformation with symmetric tensor parameter. But in this case we have *symmetric tensor parameters of rank 4 and 5*, which means that it is transformation coming from gauge field with spin 5 and 6 and our spin four gauge field participates in these transformations through the spin four gauge invariant (in zero order on field transformations) curvature.

3. Now we analyze the third line of (18) or eight terms in expression (20). First of all we see that in this remaining part of commutator our spin four field expressed through the reduced curvatures and Christoffel symbols. All such a objects possess one (first two lines of (20)) ore two (remaining two lines of (20)) pair of antisymmetrized indices contracted with composed gauge parameter. Therefore they could describe some mixed symmetry field gauge transformation acting on spin four symmetric gauge field.

This type of terms (first, third, fourth and sixth in (20)) with mixed symmetry composed parameters we can still call "regular". But four remaining terms of (20) (second, fifth, seventh and eighth) we cannot transform to regular form because they all have non contracted derivatives from one (non composed) gauge parameter ($\partial_\mu\varepsilon$ or $\partial_\mu\partial_\nu\varepsilon$) and we call these terms irregular because do not have at the moment interpretation of them in means of additional symmetries or equation of motion of theory under construction. But at least we can clime that all irregular terms are in the mixed symmetry parameter sector.

So we see that our *commutator of spin four linear on gauge field transformations produce regular terms coming from gauge transformation of symmetric tensors with spin* $s < 6$ and remaining irregular transformation with mixed symmetry gauge field parameters.

SUMMARY

In the summary, we highlighted the main results which we were able to obtain during the work, which are:

- We were able to successfully finalize the radial pullback procedure for the main term of cubic self-interaction by solving all necessary recurrence relations. The solutions of these equations lead to the possibility of obtaining the full set of AdS_{d+1} dimensional interacting terms with all curvature corrections, including trace and divergence terms from any interaction term in $d+2$ dimensional flat space.

- Using Wolfram Mathematica programming language, we developed general methods for solving recurrence relations which we faced during the radial pullback procedure. These methods are also generic enough to be used for solving different recurrence relations in other problems.

- We have considered local quartic interaction between higher-spin gauge field and scalar field. Using the full power of Neother's procedure, we successfully constructed interacting Lagrangian for this special case in a physical gauge.

- We successfully constructed and analyzed the commutator of the linearized higher spin gauge field transformation and successfully classified the right-hand side of the commutator. As an important result, we discovered that on the right-hand side of the commutator, there are terms with mixed symmetry.

- Using Wolfram Mathematica programming language, we have developed generic functions and methods which introduced the possibility of working with symmetric tensors with higher rank using the polynomial notation, which significantly makes computation much easier and human-readable.

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Կարապետյան Մելիք Մուշեղի

Փոխազդող բարձր սպինով տեսություններ հարթ և *AdS* տարածություններում

Ամփոփագիր

Բարձր սպինով տրամաչափային տեսություններն արդի տեսական ֆիզիկայի կարևորագույն բաղկացուցիչ մաս են: Այս տեսությունները պարունակում են ֆոտոնը ինչպես մեկ սպինով մասնիկ և գրավիտոնը ինչպես երկու սպինով մասնիկ: Նմանատիպ անզանգված մեկ և երկու սպինով վիճակներ գոյություն ունեն նաև լարերի տեսությունում, սակայն այնտեղ մնացած բարձր սպինով հիերարխիան ունի ոչ զրոյական զանգված, հետևաբար, բարձր սպինով տրամաչափային տեսությունները համարվում են ավելի բարձր կարգի սիմետրիայով տեսություններ, որոնցից սիմետրիայի սպոնտան խախտման միջոցով հնարավոր է տեսականորեն ստանալ լարերի տեսության բարձր սպիններով վիճակները: Տեսության այս բնագավառում կան բազմաթիվ հետաքրքիր և բարդ խնդիրներ ինչպիսին են փոխազդող բարձր սպինով տեսությունների կառուցումը, նրանց շարժման հավասարումների և լագրանժիանի ստացումը ոչ զրոյական կորությամբ ֆոնային տարածություններում:

Այս թեքնածուական ատենախոսության մեջ ներկայացված են վերոհիշյալ ուղղությունների զարգացմանը միտված արդյունքները: Թվարկենք դրանք՝

- Լուծվել են բոլոր անհրաժեշտ ռեկուրենտ հավասարումներն որոնց օգնությամբ ավարտին է հասցվել խորանարդային ինքնափոխազդեցության հիմնական անդամի ռադիալ պրոեկտումը հարթ $d+2$ չափանի տարածությունից դեպի կոր AdS_{d+1} չափանի տարածություն : Հիմնական անդամի համար կառուցվել են բոլոր AdS_{d+1} ուղղումները՝ ներառյալ հետքեր և դիվերգենցիաներ պարունակող անդամները: Արդյունքում հիմնական անդամը ներկայացվել է AdS_{d+1} օբյեկտներով և կովարիանտ ածանցյալներով: Վերոհիշյալ ռեկուրենտ հավասարումների ոչ տրիվյալ լուծումները հնարավորություն են տալիս $d+2$ չափանի հարթ տարածությունում ցանկացած գործողության համար ստանալ AdS_{d+1}տարածության մեջ այդ նույն գործողության ներկայացումը, ներառյալ հետքեր և դիվերգենցիաներ պարունակող անդամները:

- Օգտագործելով “Wolfram Mathematica” ծրագրավորման լեզուն, մշակվել են ընդհանուր մեթոդներ, որոնց միջոցով հնարավոր է եղել լուծել բարդ ռեկուրենտ հավասարումները: Վերոհիշյալ մեթոդների ունիվերսալությունը հնարավորություն է տալիս կիրառել վերջիններս այլ նմանատիպ հավասարումների լուծման համար:
- Դիտարկվել է բարձր սպինով չորրորդ կարգի լոկալ փոխազդեցություն ` բարձր սպինով դաշտի և սկալյար դաշտի միջև: Օգտագործելով Նյոտերի մեթոդը այս հատուկ դեպքի համար կառուցվել է փոխազդող լագրանժիանը` ֆիզիկական տրամաչափային պայմանի առկայության դեպքում:
- Մանրամասն ուսումնասիրվել է գծային տրամաչափային ձևափոխության կոմուտատորը, և մինչև վերջ դասակարգվել է վերջինիս հանրահաշիվը: Որպես կարևորագույն արդյունք կարելի է համարել խառը սիմետրիայի պարամետրներով ձևափոխությունների հայտնաբերումը կոմուտատորի աջ մասում:
- Օգտագործելով “Wolfram Mathematica” ծրագրավորման լեզուն, մշակվել են ընդհանուր մեթոդներ, որոնք հնարավորություն են տալիս աշխատելու բարձր սպինով օբյեկտների հետ` օգտագործելով նույն օժանդակ վեկտորային տարածություն պարունակող պոլինոմիալ նշանակումները, որոնք որ հեշտացնում են բարձր սպինով օբյեկտներ պարունակող տեսություններում հաշվարկները:

Карапетян Мелик Мушегович

Взаимодействующие теории высших спинов в плоских и AdS пространствах

Резюме

Калибровочные теориии высших спинов являются важным компонентом современной теоретической физики. Эти теории содержат фотон как частицу со спином один и гравитон — как частицу со спином два. Подобные безмассовые моды с низшими спинами существуют также в теории струн, но остальная иерархия частиц с высшими спинами имеет ненулевую массу, поэтому калибровочные теории высших спинов считаются теориями с симметрией более высокого порядка, из которых теоретически возможно получить спектр теории струн посредством спонтанного нарушения симметрии. В этой области теории есть много интересных и сложных нерешенных задач, таких как построение взаимодействующих теорий с высшими спинами, взаимодействующего лагранжиана и уравнений движения в фоновых пространствах с не нулевой кривизной.

В настоящей диссертации представлены некоторые результаты, касающиеся развития вышеуказанных направлений. А именно:

- Были решены все необходимые рекуррентные соотношения с помощью которых удалось довести до конца процедуру радиальной редукции главного члена кубического взаимодействия из $d + 2$ мерного пространства в $d + 1$ мерное пространство с постоянной кривизной (AdS_{d+1}). В результате были построены все AdS_{d+1} поправки включая следы и дивергенции. В итоге главный член взаимодействия впервые был успешно представлен напрямую в AdS пространстве на языке ковариантных производных. Нетривиальные решения рекуррентных уравнений приводят к возможности получить любое взаимодействие из плоского $d + 2$ мерного пространства в пространство Анти де Ситтера с на единицу меньшей размерности, включая члены содержащие следы и дивергенции.
- Используя среду програмирования “Wolfram Mathematica” были разработаны общие методы решения рекуррентных уравнений в отмеченной выше процедуре размерной редукции. Разработанные

нами методы настолько общие, что могут быть использованы для решения других рекурентных уравнений в смежных задачах и областях.

- Было рассмотрено взаимодействие четвертого порядка между полем с высшим спином и скаляром. Используя процедуру Нётер для этого специального случая был построен взаимодействующий Лагранжиан в физической калибровке.
- Построен и проанализирован коммутатор линеаризованного калибровочного преобразования с высшим спином. Проведена полная классификация правой части коммутатора. Как важный результат можно считать обнаружение в правой части членов соответствующих преобразованиям с параметром со смешанной симметрией.
- С помощью языка программирования Wolfram Mathematica разработаны общие методы, позволяющие работать с симметричными тензорами высокого ранга с использованием полиномиальных обозначений, содержащих то же вспомогательное векторное пространство, которое облегчает вычисления в теориях с высшими спинами.