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Գևորգ Տիգրանի Մնացականյան

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ֆիզիկամաթեմապիկական գիփությունների թեկնածուի

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Gevorg Tigran Mnatsakanyan

Estimates of sparse and Carleson type operators

SYNOPSIS

of the thesis for the degree of candidate of

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Ատենախոսության թեման հաստատվել է ՀՀ ԳԱԱ Մաթեմատիկայի Ինստիտուտում:

Գիտական ղեկավար՝	ֆ.մ.գ.դ. պրոֆեսոր Գ. Ա. Կարագուլյան
Պաշտոնական ընդդիմախոսներ՝	ֆ.մ.գ.դ. պրոֆեսոր Ք. Թիլե,
	ֆ.մ.գ.դ. պրոֆեսոր Մ. Գրիգորյան
Առաջատար կազմակերպություն՝	Հայաստանի ազգային պոլիտեխնիկական համալսարան

Պաշտպանությունը կկայանա 2022թ. հոկտեմբերի 26-ին ժ. 15:00-ին ԵՊՀ–ում գործող ԲՈԿ-ի 050 մասնագիտական խորհրդի նիստում (0025, Երևան, Ալեք Մանուկյան 1):

Ատենախոսությանը կարելի է ծանոթանալ ԵՊՀ գրադարանում:

Սեղմագիրն առաքված է 2022թ. սեպտեմբերի 6-ին:

Մասնագիտական խորհրդի գիտական քարտուղար

ֆիզ. մաթ. գիտ. դոկտոր Տ. Ն. Հարությունյան

The topic of the thesis was approved in NAS RA Institute of Mathematics.

Scientific advisor:	doctor professor G. A. Karagulyan
Official opponents:	doctor professor C. Thiele,
	doctor professor M. Grigoryan
Leading organization:	National Polytechnic University of Armenia

The defense will be held on October 26, 2022 at 15:00 at a meeting of the specialized council of mathematics 050, operating at the Yerevan State University (0025, 1 Alek Manukyan St, Yerevan).

The thesis can be found at the YSU library.

The synopsis was sent on September 6, 2022.

Scientific secretary of specialized council

doctor of phys.-math. sciences T. N. Harutyunyan

Overview

Relevance of the topic. The thesis investigates two separate questions, namely, almost everywhere convergence of the Malmquist-Takenaka series and estimates for strong-sparse operators.

The Malmquist-Takenaka (MT) series was independently introduced by Malmquist [29] and Takenaka [43] in mid 1920's and is a generalization of the classical trigonometric series. Recently, it gained importance for its connection to the non-linear phase unwinding decomposition. Let F be holomorphic in the neighborhood of the unit disk, then we consider the Blaschke factorization

$$F(z) - F(0) = B_1(z)F_1(z), \tag{1}$$

then iterating the process for F_1, one arrives at the formal series

$$F(z) = F(0) + F_1(0)B_1(z) + \cdots F_n(0)B_1(z)\dots B_n(z) + \cdots \tag{2}$$

called the non-linear unwinding decomposition. It was introduced by Nahon [33] in 2000. Numerical simulations from Nahon's doctoral thesis suggest that the unwinding decomposition converges to the initial function at an exponential rate. However, no rigorous justification of this fact is known at the moment. Main results for the unwinding decomposition was obtained in 2017 by Coifman and Steinerberger [6] who along other things proved convergence in fractional Sobolev spaces and in L^∞ for functions that are holomorphic in the neighborhood of the disk. One obtains the MT series from the unwinding decomposition by freezing the zeros of the Blaschke products. These constructions are studied by Coifman and Peyriere [5, 8]. They also proved that the MT system is an orthonormal basis in the Hardy spaces H^p, $1 < p < \infty$. The non-linear phase unwinding decomposition and the MT series are sometimes also called the adaptive Fourier transform and their theory was separately discovered and developed by Qian and others [39] who applied them to questions in Information theory. Further results on unwinding decomposition were obtained by Coifman, Steinerberger and Wu [7, 42].

From the other end, the question of almost everywhere convergence even for the classical Fourier series has been a challenging one. It was first asked by Luzin in 1920's whether the Fourier series of an L^2 function converges almost everywhere. In 1923, Kolmogorov [16] constructed an example of an L^1 function that diverges almost everywhere. Affirmative answer to Luzin's question was first obtained by Carleson in 1966 [3]. Then, Hunt [12] extended this result to L^p functions, $1 < p < \infty$, and also proved boundedness of the maximal partial sum operator, the so-called Carleson operator, on L^p. An alternative proof was given by Fefferman in 1973 [11]. However, the so-called Carleson theorem stayed an isolated result in harmonic analysis until the proof of Lacey and Thiele [20] in 2000. The proof was motivated by their work on the bilinear Hilbert transform [18, 19] and revealed the symmetry between Carleson's and Fefferman's approaches. These works stimulated active research in the next two decades in the field that has become known as time-frequency analysis. A particular instance of many generalization of the Carleson theorem that is instrumental to our work is the polynomial Carleson theorem. The polynomial Carleson operator is defined as

$$\mathcal{C}_d f(x) := \sup_{\deg P \leq d} \Big| \sup_{0<\epsilon<L<\infty} \int_{\epsilon<|y|<L} f(x-y) e^{iP(x-y)} K(y) dy \Big|, \tag{3}$$

where the first supremum is taken over all polynomial of degree at most d and K is a standard Calderón-Zygmund kernel with the corresponding operator bounded on L^2. It was conjectured by Stein in 1993 [40] that this operator is bounded on L^p. Then, Stein and Wainger [41] proved that $\mathcal{C}_d$ is bounded on L^p if the polynomials in the definition of the operator lack the linear term. This is a crucial restriction that makes the problem easier than the Carleson theorem, as the higher degree monomials make the phase highly oscillatory. The weak-L^2 estimate for $\mathcal{C}_2$ with the Hilbert kernel was obtained by Lie [26] in 2008. The full result was settled by Lie in 2011 [27, 28]. A simplification and generalization of this theorem was given by Zorin-Kranich in 2017 [44].

The sparse operators were invented by Lerner in 2012 [22, 24, 25] in connection with the so-called A_2 theorem. It is known from the works of Muckenhoupt [32], Muckenhoupt-Hunt-Wheeden [13] and Coifman-Fefferman [4] in 1970's that the Hardy-Littlewood maximal function and the Calderón-Zygmund operator is bounded on the weighted $L^p(\mathbb{R}^n, d\mu)$

spaces with a measure $d\mu$, i.e.

$$\int_{\mathbb{R}^n} |Tf|^p d\mu \leq C_{p,\mu} \int_{\mathbb{R}^n} |f|^p d\mu,$$

if and only if $d\mu = w(x)dx$ is absolutely continuous, and its density w satisfies Muckenhoupt's A_p condition

$$[w]_{A_p} := \sup_Q \left(\frac{1}{|Q|}\int_Q w\right)\left(\frac{1}{|Q|}\int_Q w^{-\frac{1}{p-1}}\right)^{p-1} < \infty. \tag{4}$$

The sharp dependence of $C_{p,d\mu}$ on the A_p characteristic of the weight for the maximal function was studied by [2] in 1993. Astala, Iwaniec and Saksman [1] raised the same question for the Ahlfors-Beurling transform, which spanned interest also for general Caldéron-Zygmund operators. The latter came to be known as the A_2 conjecture. After a number of contributions [34, 38, 37, 10, 36, 35, 17], the A_2 conjecture was solved by Hytönen in 2010 [14]. The initial proofs were based on the Bellman function technique developed by Nazarov-Treil-Volberg [34]. However, Bellman functions are only well-suited for discrete problems or continuous problems with very specific geometric symmetries. Later the approach switched to the corona decomposition which is what was used in Hytönen's proof. In 2012, Lerner [24] showed that Caldéron-Zygmund operators can be dominated in norm on on Banach lattices by rather simple sparse operators. Later Conde-Alonso, Rey [9] and Lerner, Nazarov [23] independently upgraded this result to pointwise domination. The sparse operators are useful because their weighted estimates are easy to prove. Thus, the sparse operators provided a simpler proof of the A_2 theorem. In 2017, Lacey [21] extended the previous results to domination by sparse operators of the martingale transform and Caldéron-Zygmund operators with Dini continuous kernels. Unlike the previous results, that relied on the mean oscillation formula of Lerner [22] and required the underlying measure to be doubling, Lacey's proof used only the weak-L^1 estimate of the operator. In 2019, Karagulyan [15], motivated by his work on exponential estimates of the Caldéron-Zygmund operators, introduced the concepts of the ball-basis in abstract measure spaces and of bounded oscillation (BO) operaotrs that unified the maximal function and Caldéron-Zygmund operators on spaces of homgeneous type, martingale transforms and Carleson operators. Then, in this setting he proved the same sparse domination bounds for BO operators and weighted estimates for the sparse operators.

In our work, we introduce the so-called strong-sparse operators, that are somewhat larger than the sparse operators and study their L^p bounds in abstract measure spaces with ball-basis and sharp weighted estimates in $L^2(\mathbb{R})$. We hope, that later they can be used to dominate operators, such as homogeneous rough singular integrals, that cannot be dominated by the smaller sparse operators and for which the sharp weighted bounds are not yet known.

Goals.

1. To prove L^p estimates for the maximal partial sum operators of the MT series for different configurations of the zeros of the MT system.

2. To prove weak-L^1 estimates for the strong-sparse operator on an abstract measure space with ball-basis.

3. To obtain the sharp dependence of the weighted-$L^2(\mathbb{R})$ norm of the strong-sparse operator on A_2 characteristic of the weight.

Research methods. For item 1, time-frequency methods are used along with TT^* methods. In particular, we use the methods of the polynomial Carleson theorem. We also study the behaviour of the Blaschke phases by methods of the stationary phase.

For item 2, we use Cladéron-Zygmund decomposition-type techniques to reduce the weak-L^1 bound to the strong-L^2 bound.

For item 3, we use the techniques and known results of the sparse operators and maximal functions, as well as, the main properties of the A_p weights.

Scientific novelty. All results of the thesis are new.

A connection between non-linear phase unwinding decomposition and the polynomial Carleson theorem is established. The methods of the latter are extended to include more general phase functions.

To prove the sharpness of the weighted bound of the strong-sparse operator, a new A_2 weight is constructed that is a lacunary mixture of power weights.

Theoretical and practical value. The thesis is a theoretical investigation. However, the results and methods of the MT series can be extended to be of important practical value due to their connection to Information theory.

The A_2 weight constructed for the strong-sparse operator is new in the literature and can potentially be applied to other situations.

Publications. The main results of the thesis are published in the author's works [15, 31, 30].

Structure and volume of the thesis. The thesis consists of an introduction, two chapters, a conclusion and references. 78 pages, no figures.

The content of the work

Introduction contains a historical overview of the problems, the definitions of the objects of study and statement of the main theorems.

In the first chapter we study almost everywhere convergnece of the MT series. Let $(a_n)_{n=1}^{\infty}$ be as sequence of points inside the unit disk with

$$\sum_n (1 - |a_n|) = +\infty. \tag{5}$$

Then, we define the corresponding Blaschke products an the MT system as

$$B_n(z) := \prod_{j=1}^{n} \frac{z - a_j}{1 - \overline{a_j} z}, \qquad \phi_n(z) := B_n(z) \frac{\sqrt{1 - |a_{n+1}|^2}}{1 - \overline{a_{n+1}} z}. \tag{6}$$

Coifman and Peyriere [5] proved that, for $1 < p < \infty$, $(\phi_n)_{n \geq 0}$ is an orthonormal basis in the Hardy space H^p, i.e.

$$\sum_{n=0}^{N} \langle f, \phi_n \rangle \phi_n \xrightarrow[N \to \infty]{} f \text{ in } H^p. \tag{7}$$

We are interested whether (7) holds almost everywhere. By standard techniques, an affirmative answer to this question would follow from L^p estimates for the maximal partial sum operator defined by

$$Tf(e^{ix}) := T^{(a_n)} f(e^{ix}) := \sup_n \Big| \sum_{n=0}^{N} \langle f, \phi_n \rangle \phi_n(e^{ix}) \Big|. \tag{8}$$

Our main results for this chapter are the following two theorems.

Theorem A *Let $0 < r < 1$ and let $(a_n)_{n=1}^{\infty}$ be an arbitrary sequence such that $|a_n| \leq r$ for all n. Then, for $1 < p < 2$,*

$$\|T^{(a_n)}\|_{L^p(\mathbb{T}) \to L^p(\mathbb{T})} \lesssim_p \log \frac{1}{1 - r}. \tag{9}$$

For $2 \leq p < \infty$, we have the better estimate

$$\|T^{(a_n)}\|_{L^p(\mathbb{T}) \to L^p(\mathbb{T})} \lesssim_p \sqrt{\log \frac{1}{1 - r}}. \tag{10}$$

Furthermore, for $1 < p \leq 2$, we have a lower bound in the sense, that for every $0 < r < 1$ there exists a sequence (a_n) with $|a_n| \leq r$ such that

$$\|T^{(a_n)}\|_{L^p(\mathbb{T}) \to L^p(\mathbb{T})} \gtrsim \sqrt{\log \frac{1}{1 - r}}. \tag{11}$$

In particular, the bounds (10) and (11) are sharp for $p = 2$.

Theorem B *Let a_n be inside the triangle with vertices $(1,0), (\frac{1}{2},\frac{1}{2})$ and $(\frac{1}{2},-\frac{1}{2})$ for all n, then*

$$\|T\|_{L^2(\mathbb{T})\to L^2(\mathbb{T})} \lesssim 1. \tag{12}$$

In the second chapter we study estimates for the strong-sparse operator. A collection $\mathcal{S}$ of balls in $\mathbb{R}^n$ or, in abstract measure space with ball-basis, is called (γ-)sparse if for any $B \in \mathcal{S}$ there is a set $E_B \subset B$ such that $|E_B| \geq \gamma|B|$ and the sets $\{E_B : B \in \mathcal{S}\}$ are disjoint, where $0 < \gamma < 1$ is a constant. Given a sparse collection $\mathcal{S}$, we define the associated sparse and strong-sparse operators $\mathcal{A}_{\mathcal{S}}, \mathcal{A}^*_{\mathcal{S}} : L^1_{loc}(\mathbb{R}^n) \to L^0(\mathbb{R}^n)$ by

$$\mathcal{A}_{\mathcal{S}} f(x) := \sum_{B\in\mathcal{S}} \frac{1}{|B|}\int_B |f| \cdot \mathbf{1}_B(x), \quad \mathcal{A}^*_{\mathcal{S}} f(x) := \sum_{B\in\mathcal{S}} M_B f \cdot \mathbf{1}_B(x), \tag{13}$$

where $M_B f := \sup\limits_{A\supset B \text{ balls}} \frac{1}{|A|}\int_A |f|$.

Our main results for this chapter are the following four theorems.

Theorem C *A strong-sparse operator $\mathcal{A}^*_{\mathcal{S}}$ corresponding to a general ball-basis in an abstract measure space is a bounded operator on L^p for $1 < p < \infty$, and satisfies the weak-L^1 estimate. That is*

$$\|\mathcal{A}^*_{\mathcal{S}}(f)\|_p \lesssim \|f\|_p, \quad 1 < p < \infty, \tag{14}$$

$$\mu\{\mathcal{A}^*_{\mathcal{S}}(f) > \lambda\} \lesssim \frac{\|f\|_1}{\lambda^1}, \quad \lambda > 0. \tag{15}$$

Theorem D *For an A_2 weight w we have the bound*

$$\|\mathcal{A}^*_{\mathcal{S}}\|_{L^2(w)\to L^{2,\infty}(w)} \lesssim [w]_{A_2}^{\frac{3}{2}}. \tag{16}$$

The inequality is sharp in the following sense: there exist a sparse family $\mathcal{S}$ and a sequence of weights w_α such that

$$[w_\alpha]_{A_2} \to \infty, \ \text{as } \alpha \to 0, \tag{17}$$

and for any function $\phi : [0,\infty) \to [0,\infty)$ with $\phi(x)/x^{\frac{3}{2}} \to 0$ as $x \to \infty$, we have

$$\frac{\|\mathcal{A}^*_{\mathcal{S}}\|_{L^2(w_\alpha)\to L^{2,\infty}(w_\alpha)}}{\phi([w_\alpha]_{A_2})} \to \infty, \ \text{as } \alpha \to 0. \tag{18}$$

Theorem E *For an A_2 weight w we have the bound*

$$\|\mathcal{A}_{\mathcal{S}}^*\|_{L^2(w)\to L^2(w)} \lesssim [w]_{A_2}^2. \tag{19}$$

The inequality is sharp in the following sense: there exist a sparse family $\mathcal{S}$ and a sequence of weights w_α such that

$$[w_\alpha]_{A_2} \to \infty, \ \ \textit{as } \alpha \to 0, \tag{20}$$

and for any function $\phi : [0,\infty) \to [0,\infty)$ with $\phi(x)/x^2 \to 0$ as $x \to \infty$, we have

$$\frac{\|\mathcal{A}_{\mathcal{S}}^*\|_{L^2(w_\alpha)\to L^2(w_\alpha)}}{\phi([w_\alpha]_{A_2})} \to \infty, \ \ \textit{as } \alpha \to 0. \tag{21}$$

Theorem F *Let the sparse family $\mathcal{S}$ be such that for any two $A, B \in \mathcal{S}$ either $A \subset B$ or $B \subset A$. Then, we have*

$$\|\mathcal{A}_{\mathcal{S}}^*\|_{L^2(w)\to L^2(w)} \lesssim [w]_{A_2}^{\frac{3}{2}}. \tag{22}$$

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1. Grigori Karagulyan and Gevorg Mnatsakanyan, *On a weak type estimate for sparse operators of strong type*, Journal of Contemporary Mathematical Analysis (Armenian Academy of Sciences) **54** (2019), 216-221.

2. Gevorg Mnatsakanyan, *On almost-everywhere convergence of Malmquist-Takenaka series*, Journal of Functional Analysis **282** (2022), no. 12, 109461.

3. Gevorg Mnatsakanyan, *Sharp weighted estimates for strong-sparse operators*, Journal of Contemporary Mathematical Analysis **57** (2022), no. 4, 222-231.

Ամփոփում

Ատենախոսությունում ստացվել են հետևյալ հիմնական արդյունքները.

- Մալմքուիստ-Տակենակայի շարքի մասնակի գումարների մաքսիմալ օպերատորի համար ստացվել են L^p, $1 < p < \infty$, գնահատականներ, երբ համակարգը ծնող հաջորդականությունը գտնվում է միավոր շրջանի ներսի կոմպակտ շրջանում: Ընդ որում՝ օպերատորի L^2 նորմը սահմանափակ է կոմպակտի հիպերբոլիկ տրամագծի քառակուսի արմատով, և այս գնահատականը ճշգրիտ է:

- Գնդային բազիսով օժտված չափով աբստրակտ տարածություններում որոշված ուժեղ-նոսր տիպի օպերատորների համար ստացվել են թույլ $(1,1)$ և ուժեղ (p,p) տիպի գնահատականներ:

- Իրական առանցքի վրա որոշված ուժեղ-նոսր տիպի օպերատորների համար ստացվել են ճշգրիտ գնահատականներ կշռային L^2 և կշռային թույլ L^2 տարածություններում: Մասնավորապես՝ կառուցվել է A_2 կշիռ, որը հանդիսանում է համալուծ աստիճանային կշիռների խառնուրդ և նոր է գրականության մեջ:

Заключение

В диссертации были получены следующие основные результаты:

- Для максимального оператора частичных сум Малмквиста-Такенаки получены оценки в пространствах L^p, $1 < p < \infty$, когда рождающая систему последовательность точек находится в компактной окружности внутри единочной окружности. Более того, показно, что L^2 норма оператора оценивается квадратным корнем от гиперболического диаметра компакта и эта оценка является точной.

- Для сильных-sparse операторов, определенных в абстрактном пространстве с мерой, получены оценки слабого типа $(1,1)$ и сильного типа (p,p).

- Для сильных-sparse операторов, определенных на вещественной прямой, получены точные оценки в взвешанной L^2 и взвешанной слабой L^2 пространствах. В частности, строится A_2 вес, каторый является смесью сопряженных степенных весов и является новой в литературе.