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Report on the thesis
"Uniqueness and reconstruction for orthogonal spline systems"

It is my pleasure to give a report on the thesis "Uniqueness and reconstruction for orthogonal spline systems", submitted by Ani Levon Khachatryan in fulfillment of the requirement for the PhD in Physical and Mathematical Sciences. Specialization: A.01.01-Mathematical Analysis.
The thesis at hand consists of three chapters, based on the three works [42,43,44] by the author as referenced in the thesis. It considers sufficient and necessary conditions under which the coefficients of certain orthogonal spline series can be recovered by their limiting functions. Generally those results have the following form:
Assume that an orthogonal spline series $\sum_n a_n f_n$ converges in measure to some limiting function f and assume that for some increasing sequence (n_j) of natural numbers, the corresponding maximal function $M = \sup_j \left|\sum_{i\le n_j} a_i f_i\right|$ satisfies $\lambda_m \cdot \{M > \lambda_m\} \to 0$ as $\lambda_m \to \infty$.
Then, each coefficient a_n is given by

$$a_n = \lim_{m\to\infty} \int [f(x)]_{\lambda_m} f_n(x)\,dx, \tag{1}$$

with the truncation $[y]_t = y$ if $y \le t$ and $[y]_t = 0$ if $y > t$.
In particular, Chapter 1 extends a result by K. Keryan [26] which considers the dyadic Franklin system (f_n) (piecewise linear), certain piecewise constant functions λ_m tending to ∞ and $n_j = 2^j$ to more general subsequences (n_j) with the only restriction that the ratio n_j/n_{j-1} is bounded from above.
Chapter 2 extends a result by G. Gevorkyan and M. Poghosyan [18] that considers tensor products of dyadic Franklin functions (f_n), $n_j = 2^j$ and a real sequence $\lambda_m \to \infty$ to the case where tensors of orthogonal spline systems of higher order are considered.
Chapter 3 investigates the univariate setting and considers more general point sequences and corresponding orthogonal spline systems of arbitrary order. Here, it is proved that two certain regularity conditions on the partitions are sufficient to guarantee that the coefficients are recovered as in (1). Moreover, it is also shown that those two regularity conditions are also necessary to deduce (1) for each n.
The results contained in this thesis are new, interesting and are an advancement in their field. Their proofs are correct and show a remarkable command of the necessary analytical toolbox by the author. The dissertation is clearly written, well structured, and preluded by a carefully thought-out introduction that summarizes earlier works in this direction. In conclusion, it is my opinion that Ani Levon Khachatryan deserves to be awarded the PhD in Physical and Mathematical Sciences. Specialization: A.01.01-Mathematical Analysis.

Sincerely,

Markus Passenbrunner