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Report on the dissertation by Davit M. Martirosyan

titled "Investigation of Bounded Convex Bodies in R^n by Probabilistic Methods"

submitted for the degree of Candidate of Physical and Mathematical Sciences

A.01.05 – Theory of Probability and Mathematical Statistics

The dissertation is centered on exploring the interplay between geometric attributes and probabilistic characteristics of convex bodies within the context of Euclidean spaces. His research endeavors to address pivotal challenges associated with the identification and characterization of convex bodies, which represents a significant and contemporary question in the realms of integral and stochastic geometry.

In the first chapter, the author obtains the explicit distributions of chord lengths dependent on orientation for both an arbitrary convex quadrilateral and a right prism with a convex quadrilateral base. These distributions are known to uniquely characterize, within the class of convex bodies of their respective dimensions, the quadrilateral and the prism, disregarding translations and reflections. Simultaneously, explicit formulas are obtained for the covariograms of the considered bodies. In the first three sections, the focus was paid to particular prisms with rectangular and trapezoidal bases. The result presented in Theorem 1.3.1 is written in terms of extensive notations and might need a better organization. In the next section, where a general quadrilateral is considered, the author cleverly introduced the concept of a standard image to identify the figure, and then defined five new parameters, named as first- and second-order phi-diameters and three supplementary measures, closely related with the concept of X-ray (page 7). In terms of these parameters, the orientation-dependent chord length distribution function and the covariogram of an arbitrary convex quadrilateral are comfortably represented in section 1.5, through Theorems 1.5.1 and 1.5.2. The corresponding results for prisms are obtained in section 1.7, through Theorems 1.7.1 and 1.7.2. These newly designed parameters were also useful to obtain continuity criteria for the orientation-dependent chord length distribution functions for both two- and three-dimensional cases (Corollaries 1.5.1 and 1.7.1). Calculation of the first- and second-order phi-diameters as well as supplementary measures in terms of the characteristics of the standard image are separately enclosed in section 1.6. The author uses two trigonometric transformations (X_phi and L_phi) to obtain closed-form expressions for all the required quantities (Theorems 1.6.1-1.6.4).

Chapter 2 addresses the probability of n random lines intersecting at k points within a given convex domain D on the Cartesian plane. This is a classic problem in stochastic geometry with a strong geometric focus. While Rolf Sulanke successfully solved the problem for three lines, extrapolating to n lines introduces significant challenges, including the need to identify suitable geometric invariants of D. Even the step from three to four lines entails considerable complexity.

The research relies on a combinatorial algorithm by R. Ambartzumian, adapted and expanded by the author (Theorem 2.1). This approach connects the number of intersection points with specific functionals of point sets along the boundary of D. The novelty of the work is evident, and the inclusion of classic integral geometry concepts is commendable. The chapter progresses through systematic case studies, which are meticulously structured and validated (sections 2.3-2.5). The author's idea of using explicit formulas for intersection probabilities to estimate geometric invariants of D through random experiments is noteworthy. Numerical integration could be more efficient than the simulations referenced in section 2.6. This section also introduces simplified expressions for the invariants in terms of the disc's radius r (Lemma 2.6.2), with exact intersection probabilities calculated (Theorem 2.6.1). This detailed example enhances the conclusion of the chapter.

Chapter 3 explores the extension of the concept of covariogram from bounded convex bodies to the entire Euclidean space R^d The initial challenge is to define randomness in this broader context, where R^d serves as the domain. The traditional uniform distribution isn't suitable here, so the authors opt for a multivariate normal distribution. It would be interesting to explore also other distributions. It could also be valuable.

The next hurdle involves adapting geometric concepts to the infinite-dimensional setting. The newly defined "normal covariogram" addresses this by maintaining consistency with the classical covariogram, specifically in terms of the relationship between the covariogram and interpoint distances (see Definition 3.5.1, Theorem 3.5.1, Remark 3.5.1). This is achieved by studying the distribution and probability density functions of the Euclidean distance between two Gaussian points in R^d. When these Gaussian points have uncorrelated standard normal coordinates, the interpoint distance follows a generalized Gamma distribution (as established in Lemma 3.2.1 and Theorem 3.2.2). The core results generalize this finding to the case where the coordinates are correlated according to a covariance matrix. Integral representations are derived for the distribution and density functions of the Euclidean distance between such Gaussian points (Theorem 3.3.1, Corollary 3.3.1). The work is further enriched by the specific case of

R^2, where a closed-form expression for the density function is found (Theorem 3.4.1). Additionally, bounds on the moments of the Euclidean distance for any dimension d are derived in terms of the covariance matrix's extreme eigenvalues (Theorem 3.4.2).

The dissertation offers several novel insights and addresses complex technical problems. Its main results are original, clearly articulated, and precisely proven. The structure is coherent, and the writing is clear, meeting the high standards expected of a PhD thesis. The work addresses actual questions related to recognizing convex bodies through probabilistic characteristics of their lower-dimensional sections.

In summary, the dissertation titled "Investigation of Bounded Convex Bodies in R^n by Probabilistic Methods" meets the requirements set by the Armenian Higher Education and Science Committee. Its author, Davit M. Martirosyan, deserves the degree of Candidate of Physical and Mathematical Sciences, A.01.05 – Theory of Probability and Mathematical Statistics.

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